Sampling

Week 7, Fri 17 Oct 2003

• p1 demos
• sampling
News

• hw 1 solutions out
  – no more accepted as of right now

• next week
  – Mon: midterm
    • no Mon office hours, I’m away at conferences
  – Wed: Prof. van de Panne on animation
  – Fri: TA Abhijeet Ghosh on textures

• correct p1 grades posted on web site now

• project 1
  – finish hall of fame demos
Point Sampling

- multiply sample grid by image intensity to obtain a discrete set of points, or samples.
Spatial Domain

- image as spatial signal

Examples from Foley, van Dam, Feiner, and Hughes
Spatial Domain: Summing Waves

- represent spatial signal as sum of sine waves (varying frequency and phase shift)
- very commonly used to represent sound “spectrum”
Frequencies: Summing Spikes

\[ g(t) = \alpha \sin(\omega t) \]

\[ G(f) = \frac{\alpha}{2\pi} \delta(f - \frac{\omega}{2\pi}) \]

\[ g(t) = 2 \sin(\omega t) + 0.5 \sin(4\omega t) \]

\[ g(t) = \alpha \sin(\omega t) + \alpha \sin(6\omega t) \]

\[ f = \frac{\omega}{2\pi} \]

\[ f = \frac{\omega}{2\pi}, \frac{\omega}{2\pi}, \frac{\omega}{2\pi}, \frac{\omega}{2\pi} \]
Frequency Domain

- position: frequency
- height: strength of each frequency
  - sine wave: impulse
  - square wave: infinite train of impulses
Fourier Transform Example

spatial domain  

frequency domain
Sampling
Sampling Theorem

continuous-time signal can be completely recovered from its samples iff the sampling rate is greater than twice the maximum frequency present in the signal.

- Claude Shannon
Nyquist Rate

- the lower bound on the sampling rate equals twice the highest frequency component in the image’s spectrum
- this lower bound is the Nyquist Rate
Falling Below Nyquist Rate

• when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  – this is aliasing!
Flaws with Nyquist Rate

• samples may not align with peaks

Fig. 14.16 Sampling at the Nyquist rate (a) at peaks, (b) between peaks, (c) at zero crossings. ( Courtesy of George Wolberg, Columbia University.)
Nyquist Rate

\[ f_s < 2f \]

\[ f_s = 2f \]

\[ f_s > 2f \]
Nyquist and Checkerboards

- point sampled 1D checkerboard: aliases

- unweighted area sample: still have aliasing
Band-limited Signals

• if you know a function contains no components of frequencies higher than $x$
  – band-limited implies original function will not require any ideal functions with frequencies greater than $x$
  – facilitates reconstruction
  – avoids Nyquist Limit mistakes

• to lower Nyquist rate, remove high frequencies from image: *low-pass filter*
  – only low frequencies remain: band-limited
Low-Pass Filtering

Original signal

\[ \text{Low-pass filtering} \]

Low-pass filtered signal
Low-Pass Filtering

Fig. 14.20 The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)
Filtering

• low pass
  – blur

• high pass
  – edge finding
Filtering in Spatial Domain

- blurring or averaging pixels together

\[ h(x) = f \otimes g = \int f(x)g(x-y)dy \]
Filtering in Frequency Domain

- multiply signal’s spectrum by pulse function
Common Filters

Spatial domain vs. frequency domain
Dualities

- inverse relationship between size
  - $T$ large $\rightarrow 2\pi/T$ small

Spatial domain

Frequency domain

$s(x)$ $\rightarrow$ $S(u)$
Sinc Function

• sinc (pulse) function is common filter:
  – $sinc(x) = \frac{\sin(\pi x)}{\pi x}$
  – infinite in frequency domain
Sampling in Spatial Domain

• Q: what is sampling (i.e. evaluating a continuous function at evenly spaced points)?
• A: multiplication of the sample with a regular train of delta functions (spikes).
Sampling in Frequency Domain

- multiple copies of spectrum
- example: given spectrum $S(\omega)$ of a signal $s(t)$
Sampling in Frequency Domain

• multiple shifted copies of $S(\omega)$ are added up during sampling

• if $2\pi/T$ is large enough ($T$ is small enough)
  – individual spectrum copies do not overlap
  – *depends on maximum frequency* $\omega_0$ in $s(t)$
Sampling in Frequency Domain

- if $T$ is too large ($2\pi/T$ is small), overlap occurs
  - this is aliasing
Undersampling leads to aliasing.

Samples are too close together in f.

Spurious components:
Cause of aliasing.
How do we remove aliasing?

- perfect solution - prefilter with perfect bandpass filter.
How do we remove aliasing?

- perfect solution - prefilter with perfect bandpass filter.
  - difficult/Impossible to do in frequency domain
- convolve with sinc function in space domain
  - optimal filter - better than area sampling.
  - sinc function is infinite !!
  - computationally expensive
How do we remove aliasing?

• cheaper solution: take multiple samples for each pixel and average them together → supersampling.
• can weight them towards the centre → weighted average sampling
• stochastic sampling
• importance sampling

Removing aliasing is called antialiasing
Weighted Sampling

- multiple samples per pixel

![Diagram of weighted sampling with 3x3 and 5x5 Bartlett weights]

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<th>3x3 Bartlett</th>
<th>5x5 Bartlett</th>
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<tr>
<td></td>
<td>1 2 3 2 1</td>
</tr>
</tbody>
</table>
Stochastic Supersampling

• high frequency noise preferable to aliases
Importance Sampling

equal distribution
unequal weights

unequal distribution
equal weights