News

- Office hours reminder: FSC 2618
  - Mondays 10:30-11:30 or by appointment
  - exceptions: Oct 20, Nov 10

- Readings
  - Chap 8.9-8.11, Fri 10/3 slide notes

Transforming Normals

- nonuniform scaling does not work
  - line x=y
  - normal: [1,-1,0]
  - ignore normalization for now

Transforming Normals

- apply nonuniform scale: stretch along x by 2
  - new plane x = 2y
- transformed normal

News

- project 1
  - solution today
  - hall of fame next week
    - great work!!
- extra office hours
  - Fri 10-11 usual, 11-1:30 extra lab hours
  - Mon 10/13 no class, no office hours
  - Tue 11-1 extra lab hours,
    - 4-5:30 my office hours rescheduled (FSC2618)

Barycentric Coordinates recap

weighted combination of vertices

\[ P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \]
\[ \alpha + \beta + \gamma = 1 \]
\[ 0 \leq \alpha, \beta, \gamma \leq 1 \]

"convex combination of points"

\[ P_1 \] (1,0,0) \[ P_2 \] (0,0,1) \[ P_3 \] (0,1,0)
Finding Correct Normal Transform

- transform a plane

\[ P' = MP \quad \text{if we know } M, \text{ what should } Q \text{ be?} \]

\[ N^T P' = 0 \quad \text{and} \quad (QN)^T (MP) = 0 \]

\[ (N^T Q^T) MP = 0 \quad \text{true if} \quad Q^T M = I \]

thus the normal to any surface has to be transformed by the inverse transpose of the modelling transformation.

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Sampling and Antialiasing

- All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.

Displays → Signal Reconstruction

Samples

- most things in the real world are continuous
- everything in a computer is discrete
- the process of mapping a continuous function to a discrete one is called sampling
- the process of mapping a discrete function to a continuous one is called reconstruction
- the process of mapping a continuous variable to a discrete one is called quantization
- rendering an image requires sampling and quantization
- displaying an image involves reconstruction

Imaging Devices Area Sample

- video camera : CCD array.

\[ V = k \int \int \text{Id} \text{d}x \text{d}y \]

Displays → Signal Reconstruction

Imaging Devices Area Sample

- eye : photoreceptors

\[ \text{Films similar to irregular array of receptors.} \]

Continuous Luminosity Signal

Sampled Luminosity

Reconstructed Luminosity

Reconstruction Artefact

Bad Solution for Jaggies

- blurring final image

Line Segments

- we tried to sample a line segment so it would map to a 2D raster display
- we quantized the pixel values to 0 or 1
- we saw stair steps, or jaggies
Line Segments

• instead, quantize to many shades
• but what sampling algorithm is used?

Area Sampling

• shade pixels according to the area covered by thickened line
• this is unweighted area sampling
• a rough approximation formulated by dividing each pixel into a finer grid of pixels

Unweighted Area Sampling

• primitive cannot affect intensity of pixel if it does not intersect the pixel
• equal areas cause equal intensity, regardless of distance from pixel center to area

Weighted Area Sampling

• unweighted sampling colors two pixels identically when the primitive cuts the same area through the two pixels
• intuitively, pixel cut through the center should be more heavily weighted than one cut along corner

Weighted Area Sampling

• weighting function, $W(x,y)$
  – specifies the contribution of primitive passing through the point $(x, y)$ from pixel center

Images

• an image is a 2D function $I(x, y)$ that specifies intensity for each point $(x, y)$
Sampling and Image

- our goal is to convert the continuous image to a discrete set of samples
- the graphics system’s display hardware will attempt to reconvert the samples into a continuous image: reconstruction

Point Sampling an Image

- simplest sampling is on a grid
- sample depends solely on value at grid points

Point Sampling

- multiply sample grid by image intensity to obtain a discrete set of points, or samples.

Sampling Errors

- some objects missed entirely, others poorly sampled

Fixing Sampling Errors

- supersampling
  - take more than one sample for each pixel and combine them
  - how many samples is enough?
  - how do we know no features are lost?

Spectral/Fourier Analysis

- spectral representation treats the function as a weighted sum of sines and cosines
- every function has two representations
  - spatial (time) domain - normal representation
  - frequency domain - spectral representation
- Fourier transform converts between the spatial and frequency domains.
Spatial Domain

- image as spatial signal

Spatial Frequency

- in time - cycles per second
- in space - cycles per meter, degree, etc.

- Fourier view: sum of signals
  - pick frequency, phase shift
  - familiar example: sound spectrum

Summing Waves I

Summing Waves II

Waves as Frequencies

Frequency Domain

- height represents strength of each frequency
  - sine wave: impulse
  - square wave: infinite train of impulses
Spectral/Fourier Analysis

- Fourier transform converts between the spatial and frequency domain
  \[ F(\omega) = \frac{1}{2\pi} \int f(x)e^{-i\omega x} dx \]
  \[ f(x) = \int F(\omega)e^{i\omega x} d\omega \]
- Euler formula: \( e^{it} = \cos t + i\sin t \)
  - real and imaginary components
- forward and reverse transforms very similar
  - reversal in sign of imaginary component, scale constant

Fourier Analysis

- convert spatial domain to frequency domain
  \[ F(u) = \frac{1}{\sqrt{2\pi}} \int f(x)\cos 2\piux - i\sin 2\piux dx, \]
  - let \( f(x) \) indicate the intensity at a location in space, \( x \) (pixel value)
  - \( u \) is a complex number representing frequency and phase shift
    - \( i = \sqrt{-1} \) ... frequently not plotted
  - \( F(u) \) is the amplitude of a particular frequency in a signal
    - in this case the signal is \( f(x) \)

Sampling Theorem

The ideal samples of a continuous function contain all the information in the original function if and only if the continuous function is sampled at a frequency greater than twice the highest frequency in the function

- Claude Shannon

Nyquist Rate

- the lower bound on the sampling rate equals twice the highest frequency component in the image’s spectrum
- this lower bound is the Nyquist Rate
Falling Below Nyquist Rate

• when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  – this is aliasing!

Band-limited Signals

• if you know a function contains no components of frequencies higher than \( x \)
  – band-limited implies original function will not require any ideal functions with frequencies greater than \( x \)
  – facilitates reconstruction
  – avoids Nyquist Limit mistakes

Falling Below Nyquist Rate

• when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  – safe with band-limits, guarantee that samples are not derived from signal of higher frequency

Flaws with Nyquist Rate

• samples may not align with peaks

Filtering

• low pass

• high pass

Filtering

• to lower Nyquist rate, remove high frequencies from image: *low-pass filter*
  – only low frequencies remain: band-limited
Filtering in Space Domain
• blurring or averaging pixels together.

\[ h(x) = f \otimes g = \int f(x)g(x-y)dy \]

Filtering in Frequency Domain
• multiply signal’s spectrum by pulse function

Filtering
• sinc (pulse) function is common filter:
  \[ \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \]

Sinc Filter
• Slide filter along spatial domain and compute new pixel value that results from convolution

Convolutions
• multiplying two Fourier Transforms \( F(u)G(u) \) in the frequency domain \( \Rightarrow \) convolution (represented as *) on their inverse Fourier transforms in the spatial domain
  \[ f(x) \ast g(x) = h(x) \]
  - take the filter function, \( g(x) \) and center it at \( x \)
  - take a weighted average of \( f(x) \) in the neighborhood of \( x \)
    • weighting defined by \( g(x) \)

Sampling in Frequency Domain
• remember, sampling was defined as multiplying a grid of delta functions by the continuous image
• called a convolution in spatial domain

Sampling Grid
• function being sampled
**Sampling**

- Multiplication of the sample with a regular train of delta functions.

**Convolution**

- This amounts to accumulating copies of the function’s spectrum sampled at the delta functions of the sampling grid.

**Convolution theorem.**

- *Theorem:* Multiplication in the frequency domain is equivalent to convolution in the space domain.

- *Symmetric Theorem:* Multiplication in the space domain is equivalent to convolution in the frequency domain.

**Bilinear Filter**

- sometimes called a tent filter
- easy to compute
  - just linearly interpolate between samples
- finite extent and no negative values
- still has artifacts

**Sampling Pipeline**

- Original signal
- Low-pass filtered signal
- Sampling
- Reconstructed signal