Logical Generative Models for Probabilistic Reasoning about Existence, Roles and Identity

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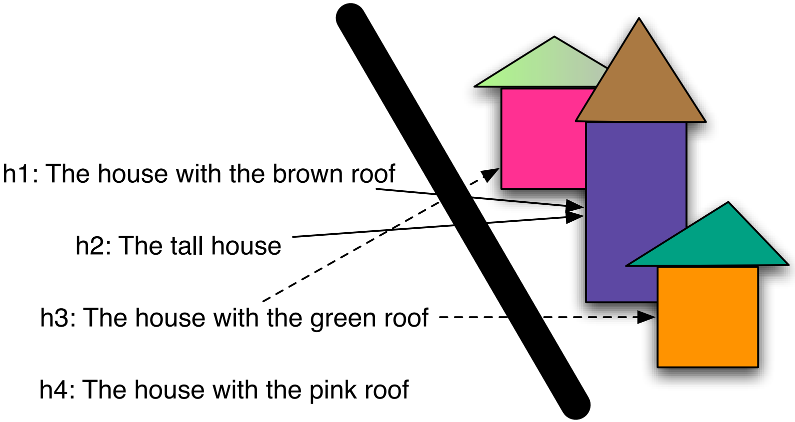
Outline

Provide a clean semantic framework for reasoning about uncertainty in existence and identity.

- Existence and Identity
- Semantic Trees
- First-order Semantic Trees
- Exchangeability
- Conclusion and future work

Existence and Identity



Clarity Principle

Clarity principle: probabilities must be over well-defined propositions.

- What if an object doesn't exist?
  - $house(h4) \wedge roof\_colour(h4, pink) \wedge \neg exists(h4)$

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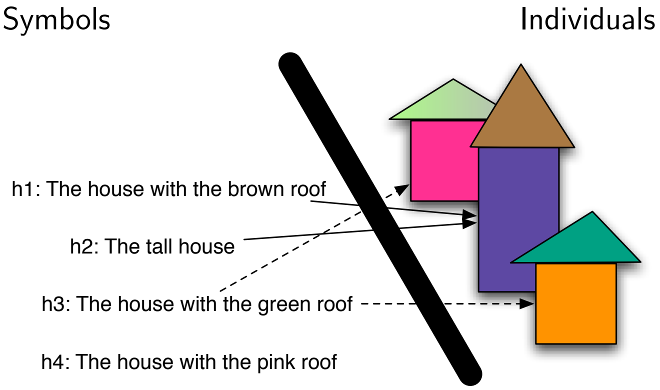
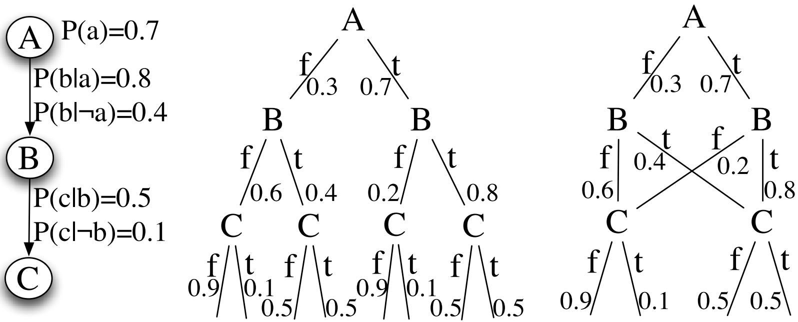
- What if an object doesn't exist?
  - $house(h4) \wedge roof\_colour(h4, pink) \wedge \neg exists(h4)$
- What if more than one object exists? Which one are we referring to?
  - In a house with three bedrooms, which is the second bedroom?

Correspondence Problem

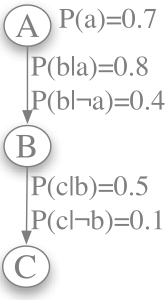


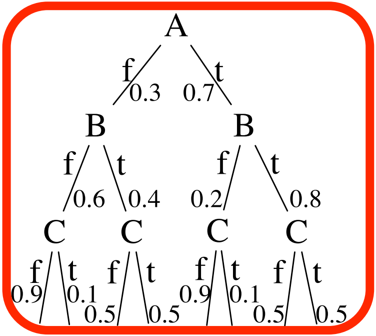
c symbols and i individuals $\longrightarrow$ c^{i+1} correspondences

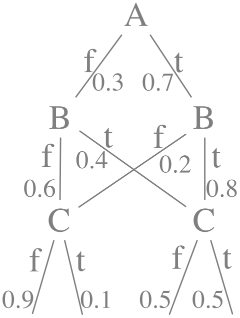
Semantic Tree



Semantic Tree







↑
semantic tree
event tree
decision tree. . .

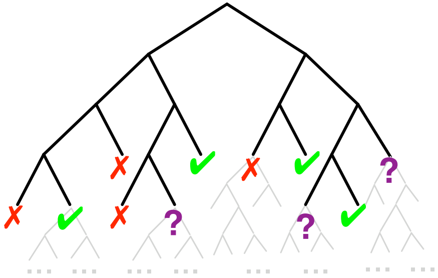
Semantic tree

- Nodes are propositions
- Left branch is when proposition is false
  Right branch is when proposition is true
- There is a probability distribution over the children of each node
- Each finite path from the root corresponds to a formula
- Each finite path from the root has a probability that is the product of the probabilities in the path

A generative model generates a semantic tree.

Infinite Semantic Tree

Given a proposition α:

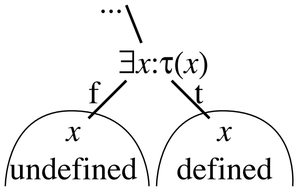


✔ *path* $\models \alpha$
✗ *path* $\models \neg\alpha$
? otherwise

The probability of α is well defined if for all $\epsilon > 0$ there is a finite sub-tree that can answer α in $> 1 - \epsilon$ of the probability mass.

First-order Semantic Trees

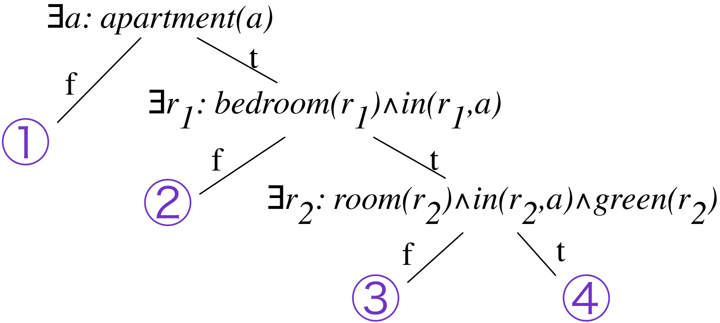
You can split on quantified first-order formulae:



- The "true" sub-tree is in the scope of x
- The "false" sub-tree is not in the scope of x

A logical generative model generates a first-order semantic tree.

First-order Semantic Tree (cont)



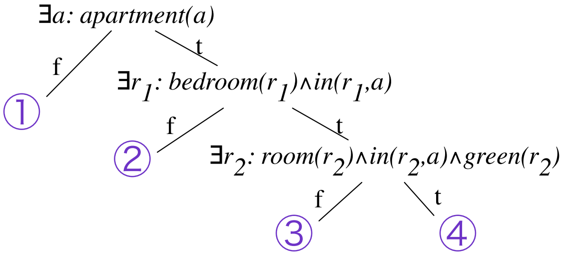
1. there is no apartment
2. there is no bedroom in the apartment
3. there is a bedroom but no green room
4. there is a bedroom and a green room

Semantics

Each path from the root corresponds to a logical formula. The **path formula** to node n is:

- The path formula of the root node is "*true*".
- If the path formula of node n is formula f and node n is labelled with formula f'
  - the "true" child of node n has path formula
    $$f \wedge f'$$
    where f' is in the scope of the quantification of f.
  - The "false" child of node n has path formula:
    $$f \wedge \neg(f \wedge f')$$

First-order Semantic Tree (cont)



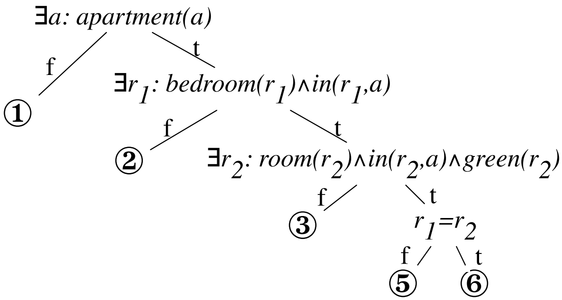
Path formulae:

① $(\neg\exists a\ apt(a))$

② $\exists a\ apt(a) \wedge \neg(\exists a\ apt(a) \wedge \exists r_1\ br(r_1) \wedge in(r_1, a))$

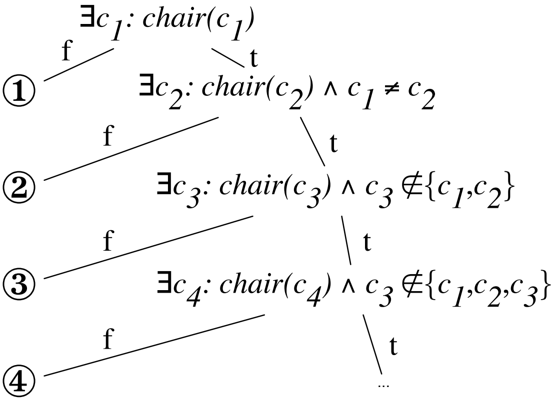
④ $\exists a\ apt(a) \wedge \exists r_1\ br(r_1) \wedge in(r_1, a) \wedge \exists r_2\ room(r_2) \wedge in(r_2, a) \wedge green(r_2)$

First-order Semantic Tree (cont)

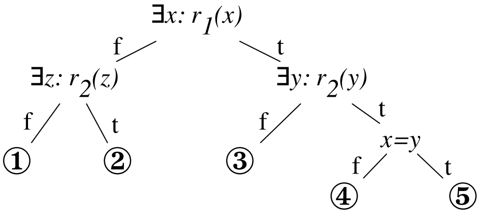


- ⑥ $\exists a\ apt(a) \wedge \exists r_1\ br(r_1) \wedge in(r_1, a) \wedge \exists r_2\ room(r_2) \wedge in(r_2, a) \wedge green(r_2) \wedge r_1 = r_2$
  There is a green bedroom.
- ⑤ There is a bedroom and a green room, but no green bedroom.

Distributions over number



Roles and Identity (1)



① there no object filling either role

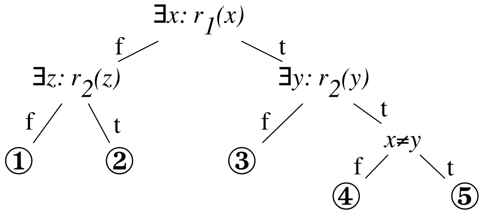
② there is an object filling role r_2 but none filling r_1

③ there is an object filling role r_1 but none filling r_2

④ only different objects fill roles r_1 and r_2

⑤ some object fills both roles r_1 and r_2

Roles and Identity (2)



① there no object filling either role

② there is an object filling role r_2 but none filling r_1

③ there is an object filling role r_1 but none filling r_2

④ only the same object fill roles r_1 and r_2

⑤ there are different objects that fill roles r_1 and r_2

Exchangeability

We can solve many probabilistic queries, but we can't draw balls out of urns!

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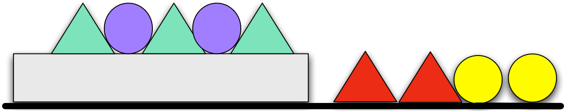
$$P(h|e) = \frac{P(h \wedge e)}{P(e)}$$

What if h refers to an object in e?

Exchangeability



Consider the query:

$$P(green(x) \mid \exists x \; triangle(x) \wedge \exists y \; circle(y) \wedge touching(x, y))$$

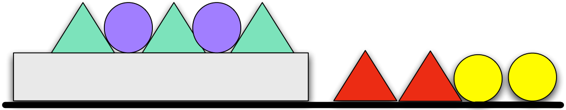
The answer depends on how the x and y were chosen!

Exchangeability

- Exchangeability: a priori each individual is equally likely to be chosen.
- A generalized first-order semantic tree is a first-order semantic tree that can contain $commit(\overline{x})$ nodes.
  For each $commit(\overline{x})$ node:
  - $\overline{x}$ is a set of variables
  - the node is in the scope of each x in $\overline{x}$
  - no x is in an ancestor commit.
  - This node has one child.

  For each possible world, each tuple of individuals that satisfies the path formula to $commit(\overline{x})$ has an equal chance of being chosen.

Commit



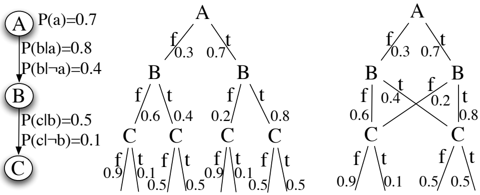
$$P(green(x) \mid \exists x \; triangle(x) \land \exists y \; circle(y) \land touching(x, y))$$

$commit(x)$	$commit(y)$	$commit(x, y)$
$commit(y)$	$commit(x)$	
$3/4$	$2/3$	$4/5$

Conclusion

- Probabilities are only over well-defined probabilities.
- We don't need to consider correspondences between symbol and objects: only between symbols
- "Only" a decision problem down each branch (except for "commit").

To Do



- A language to generate semantic trees as needed.
- Efficient inference.
- Learning the probabilities of existence and identity.
- Incorporation into existing and new frameworks...