CPSC 502 — Fall 2006

Assignment 4

Due: 12:30pm, Monday 30 October 2006.

Question 1

Consider the domain of house plumbing represented in the diagram of Figure 1. This is a simplified model of the same domain used in the previous assignment.

In this figure, $p1$, $p2$ and $p3$ denote cold water pipes. $p1$ is the pipe coming in from the main water supply. $t1$, $t2$ and $t3$ are taps and $d1$, $d2$ and $d3$ are drainage pipes. The constants *shower* denotes a shower, *bath* denotes a bath, *sink* denotes a sink, and *floor* denotes the floor. There can also be plugs in the sink or in the bath. You can assume that you are in a static situation (i.e., you don't have to worry about time); imagine that you have stumbled in this situation and must reason about it.

Suppose you can observe or query the tap positions, flow out of drain d1, and whether there is water in the sink and bath, and whether there is water on the floor.

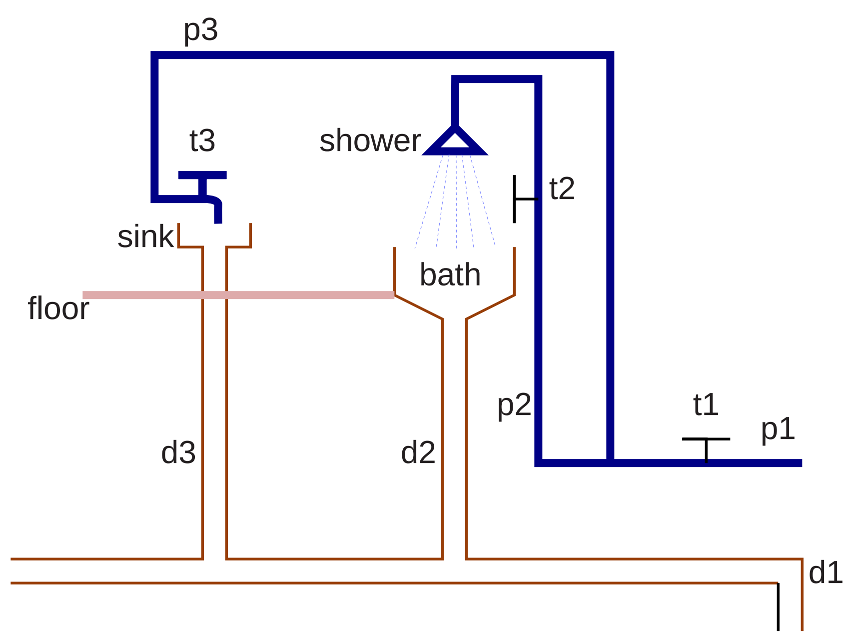


Figure 1: The Plumbing Domain

(a) What are the random variables? In particular, you need random variables to represent the observations you may want to make, the queries you may be interested and other (hidden) variables to keep the model simple. For each variable, give its domain and intended interpretation.

(b) Give a belief network for these variables, assuming a causal ordering of the variables. Give reasonable conditional probability tables. You can use the CIspace applet or any other belief network tool.

(c) Suppose you had to demonstrate to a skeptic that Belief networks are appropriate for this domain. Argue that this is a reasonable representation, and as part of your explanation, you need to give some test cases that show what your model is capable of. Use proper English. Be brief and concise (as this skeptic doesn't have much time).

(d) What would your belief network look like if you had chosen the opposite ordering of variables?

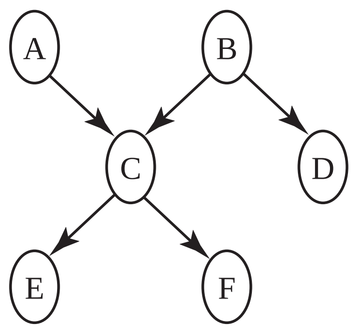
Solution

(a), (b) see http://www.cs.ubc.ca/spider/poole/cs502/2006/as4/plumbing.xml for a CIspace representation.

(d) there are many more arcs, in particular there are arcs between nodes X and Y in the new ordering if there were arcs in the original network or if they have common ancestors in the original (causal) belief network that are lower in the total ordering than both X and Y or where there are common descendents (in the original graph) of X and Y that are also parents in the new graph. [This is a convoluted way of using D-separation in the original causal network to determine parents in the inverse network.] You just need to give some examples.

Question 2

Consider the belief network:



with Boolean variables (we will write $A = true$ as a and $A = false$ as $\neg a$) and the following conditional probabilities:

$$\begin{aligned} P(a) &= 0.9 & P(d|b) &= 0.1 \\ P(b) &= 0.2 & P(d|\neg b) &= 0.8 \\ P(c|a,b) &= 0.1 & P(e|c) &= 0.7 \\ P(c|a,\neg b) &= 0.8 & P(e|\neg c) &= 0.2 \\ P(c|\neg a,b) &= 0.7 & P(f|c) &= 0.2 \\ P(c|\neg a,\neg b) &= 0.4 & P(f|\neg c) &= 0.9 \end{aligned}$$

(a) Compute $P(e)$ using variable elimination. Please try to do it by hand, then check it with the CIspace applet. Do not do any pruning. Explain the significance of each factor created (either what does it represent or why does it have particular values).

(b) Compute $P(e|\neg f)$ using variable elimination. How much of the previous computation can be reused? Show only what is different.

(c) Compute $P(a|d)$ using variable elimination.

(d) Compute $P(a|\neg f)$.

(e) Compute $P(a|\neg f, d)$.

(f) Explain when observing d affects your belief in a. (Be as specific as possible.)

Solution

Here I will not give the actual tables for the factors. Use the CIspace applet to see the tables.

(a) To compute $P(e)$, you can sum out D and F, and the factors created are just 1's (that is why they can be pruned). I'll ignore these factors.
You start off with the factors $f_0(A), f_1(B), f_2(A,B,C), f_3(C,E)$.
Eliminating A, you multiply f_0 and f_2, and sum out A, creating a factor $f_4(B,C)$.
Eliminating B, you multiply f_1 and f_4, sum out B, and create a factor $f_5(C)$.
Eliminating C, you multiply f_5 and f_3, sum out C, and the resulting factor $f_6(E)$ represents $P(E)$.

(b) To compute $P(e|\neg f)$, you create a factor (lets call it f_7 to avoid confusion with the previous part) $f_7(C)$.
You can prune D. Eliminating A and B acts exactly as before, creating the same factors.
Eliminating C, you multiply f_5, f_3 and f_7, sum out C, with the resulting factor $f_8(E)$. To get the answer, you need to divide each element of $_8$ by $\sum_E f_8(E)$. This creates $f_9(E)$ which represents $P(E|\neg f)$.

(c) Everything can be pruned! Or there are lots of tables with just ones, and a constant factor. $P(a|d) = P(a)$.

(d) E and D can be pruned.

(e) What I wanted you to notice here is that D is not irrelevant when F is also observed.

(f) When C, E or F are observed and B isn't observed.

Question 3

Suppose you want to compute $P(f|d)$ in the belief network of the previous question. You are to use this example to explain how each of the following work. (Assume you are writing a textbook for your peers, and are using this example to explain these concepts)

(a) Rejection sampling

(b) Importance sampling

(c) Particle filtering

Solution

In all of these I assume you remove E is it is irrelevant.

(a) Rejection sampling: Sample B, then sample D, then you can reject the sample if the sample doesn't assign $D = true$. Then sample the rest of the variables. Return the proportion of the samples where $F = true$.

(b) Importance sampling: Sample B. Weight those samples where $B = true$ by 0.8 and those samples with $B = false$ by 0.8. [You can actually do something smarter if you don't sample B by its prior, but use a different proposal distribution.]. Then sample the other variables. Return the weighted average of the samples with $F = true$.

(c) Particle filtering: create lots of samples (here I will use 1000 as an example). Sample B 1000 times. Weight each sample using D as in importance sampling. Now generate a new 1000 samples, choosing each sample with probability proportional to its weight. Sample the rest of the variables with these new samples, and return the proportion of the samples where $F = true$.