Minmax and Maxmin

ISCI 330 Lecture 8

February 1, 2007

Lecture Overview

1. Recap
2. Maxmin and Minmax

Computing Mixed Nash Equilibria

- Guess the support
- If a player has a support of size 2 or more, he must be indifferent between these actions
- Set up an equation that expresses these constraints:
  - e.g., $u_1(B,(p,1-p)) = u_1(F,(p,1-p))$
- Solve the equation to find p.

Lecture Overview

Max-Min Strategies

- Player i's maxmin strategy is a strategy that maximizes i's worst-case payoff, in the situation where all the other players (whom we denote $-i$) happen to play the strategies which cause the greatest harm to i.
- The maxmin value (or safety level) of the game for player i is that minimum amount of payoff guaranteed by a maxmin strategy.
- Why would i want to play a maxmin strategy?

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- Why would i want to play a maxmin strategy?
  - a conservative agent maximizing worst-case payoff
  - a paranoid agent who believes everyone is out to get him

Definition

The maxmin strategy for player i is $\arg\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$, and the maxmin value for player i is $\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$.

Min-Max Strategies

- Player i's minmax strategy in a 2-player game is a strategy that minimizes the other player $-i$'s best-case payoff.
- The minmax value of the 2-player game for player i is that maximum amount of payoff that $-i$ could achieve under i's minmax strategy.
- Why would i want to play a minmax strategy?

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- Why would i want to play a minmax strategy?
  - to punish the other agent as much as possible

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Definition

In a two-player game, the minmax strategy for player i is $\arg\min_{s_i}$ $\max_{s_{-i}} u_{-i}(s_1, s_2)$, and the minmax value for player i is $\min_{s_i}$ $\max_{s_{-i}} u_{-i}(s_1, s_2)$.

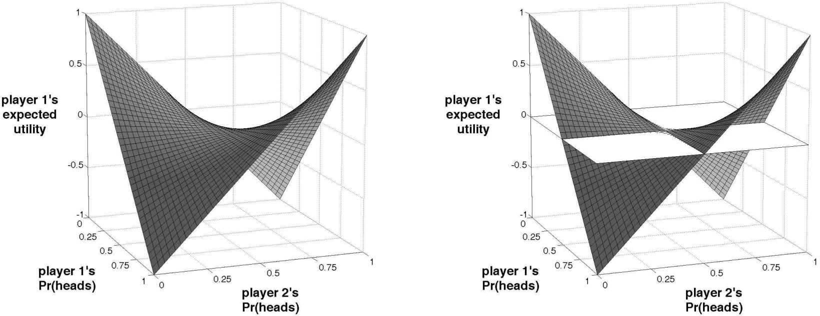
Minmax Theorem

Theorem (Minmax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

- The maxmin value for one player is equal to the minmax value for the other player. By convention, the maxmin value for player 1 is called the value of the game.
- For both players, the set of maxmin strategies coincides with the set of minmax strategies.
- Any maxmin strategy profile (or, equivalently, minmax strategy profile) is a Nash equilibrium. Furthermore, these are all the Nash equilibria. Consequently, all Nash equilibria have the same payoff vector (namely, those in which player 1 gets the value of the game).

Geometric Representation: Saddle Point



- Can you see why this picture illustrates the maxmin and minmax values?

How to find maxmin and minmax strategies

Consider maxmin strategies for player i in a 2-player game.

- Notice that i's maxmin strategy depends only on i's utilities
  - thus changes to $-i$'s utilities do not change i's maxmin strategy
- Consider the game where player i has the same utilities as before, but player $-i$'s utilities are replaced with the negatives of i's utilities
  - this is now a zero-sum game
- Because of the minmax theorem, we know that any Nash equilibrium strategy in this game is also a maxmin strategy
  - Thus, find player i's equilibrium strategy in the new game and we have i's maxmin strategy in the original game
- We can use a similar approach for minmax.