Analyzing Games: Nash Equilibrium Recap

ISCI 330 Lecture 6

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Lecture Overview

What are solution concepts?

- Solution concept: a subset of the outcomes in the game that are somehow interesting.
- There is an implicit computational problem of finding these outcomes given a particular game.
- Depending on the concept, existence can be an issue.

Solution concepts we've seen so far:

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Solution concepts we've seen so far:

- Pareto-optimal outcome
- Pure-strategy Nash equilibrium
- Mixed-strategy Nash equilibrium
- Other Nash variants:
    - weak Nash equilibrium
    - strict Nash equilibrium

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy s_i for agent i as any probability distribution over the actions A_i.
    - pure strategy: only one action is played with positive probability
    - mixed strategy: more than one action is played with positive probability
        - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be S_i
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$.

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile $s \in S$?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

- Best response:
  - $s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i$, $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$
- Nash equilibrium:
  - $s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i$, $s_i \in BR(s_{-i})$
- Every finite game has a Nash equilibrium! [Nash, 1950]
  - e.g., matching pennies: both players play heads/tails 50%/50%

Lecture Overview

Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy? Different interpretations:

- Randomize to confuse your opponent
  - consider the matching pennies example
- Players randomize when they are uncertain about the other's action
  - consider battle of the sexes
- Mixed strategies are a concise description of what might happen in repeated play: count of pure strategies in the limit
- Mixed strategies describe population dynamics: 2 agents chosen from a population, all having deterministic strategies. MS is the probability of getting an agent who will play one PS or another.