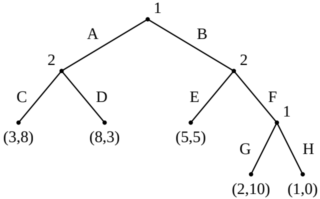
Backward Induction

ISCI 330 Lecture 14

March 1, 2007

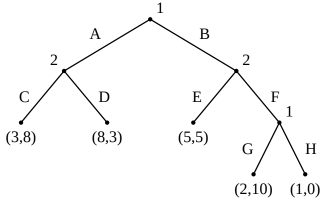
Lecture Overview

Subgame Perfection



- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him

Subgame Perfection



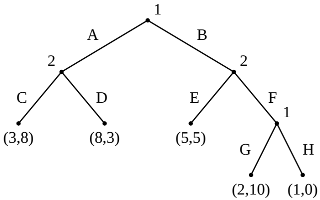
- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him
  - He does it to threaten player 2, to prevent him from choosing F, and so gets 5
    - However, this seems like a non-credible threat
    - If player 1 reached his second decision node, would he really follow through and play H?

Formal Definition

- Define subgame of G rooted at h:
  - the restriction of G to the descendents of H.
- Define set of subgames of G:
  - subgames of G rooted at nodes in G

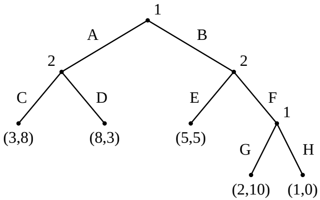
- s is a subgame perfect equilibrium of G iff for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'
- Notes:
  - since G is its own subgame, every SPE is a NE.
  - this definition rules out "non-credible threats"

Back to the Example



- Which equilibria from the example are subgame perfect?

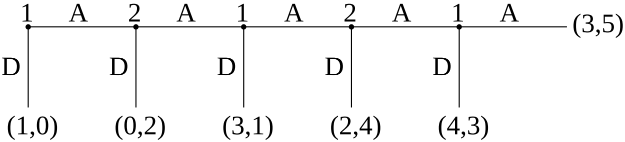
Back to the Example



- Which equilibria from the example are subgame perfect?
  - $(A, G), (C, F)$ is subgame perfect
  - (B, H) is an non-credible threat, so $(B, H), (C, E)$ is not subgame perfect
  - (A, H) is also non-credible, even though H is "off-path"

Lecture Overview

Centipede Game

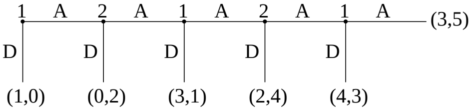


- Play this as a fun game...

Computing Subgame Perfect Equilibria

- Idea: Identify the equilibria in the bottom-most trees, and adopt these as one moves up the tree
- Procedure
  - starting at each terminal node, move up to its parent node and label it with the utility values of the terminal node that would be the best response for the player who gets to choose at this parent node
  - repeat this procedure, treating the highest already labeled nodes as terminal nodes, until the root is reached
  - Stop when the root of the tree is labeled
  - Note: Before performing this procedure at any given node, it must be performed at all subnodes first
- the procedure doesn't return an equilibrium strategy, but rather labels each node with a vector of real numbers.
  - This labeling can be seen as an extension of the game's utility function to the non-terminal nodes
  - The equilibrium strategies: take the best action at each node.

Backward Induction



- What happens when we use this procedure on Centipede?
  - In the only equilibrium, player 1 goes down in the first move.
  - However, this outcome is Pareto-dominated by all but one other outcome.
- Two considerations:
  - practical: human subjects don't go down right away
  - theoretical: what should you do as player 2 if player 1 doesn't go down?
    - SPE analysis says to go down. However, that same analysis says that P1 would already have gone down. How do you update your beliefs upon observation of a measure zero event?
    - but if player 1 knows that you'll do something else, it is rational for him not to go down anymore... a paradox
    - there's a whole literature on this question