Extensive Form Games and Backward Induction

ISCI 330 Lecture 13

February 27, 2007

Lecture Overview

Recap

Subgame Perfection

Backward Induction

Nash Equilibria

Given our new definition of pure strategy, we are able to reuse our old definitions of:

- mixed strategies
- best response
- Nash equilibrium

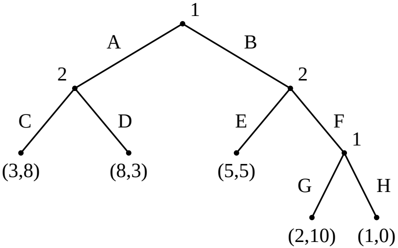
Theorem

Every perfect information game in extensive form has a PSNE

This is easy to see, since the players move sequentially.

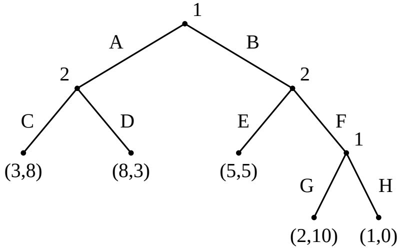
Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can “convert” an extensive-form game into normal form



Induced Normal Form

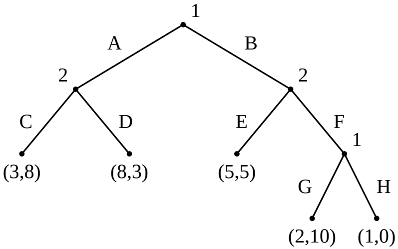
- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form



	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form

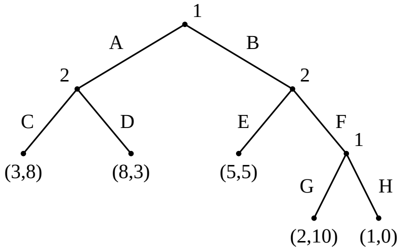


	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

- this illustrates the lack of compactness of the normal form
  - games aren't always this small
  - even here we write down 16 payoff pairs instead of 5

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form

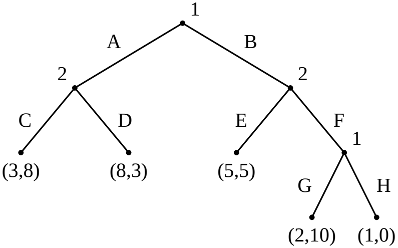


	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

- while we can write any extensive-form game as a NF, we can't do the reverse.
  - e.g., matching pennies cannot be written as a perfect-information extensive form game

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form

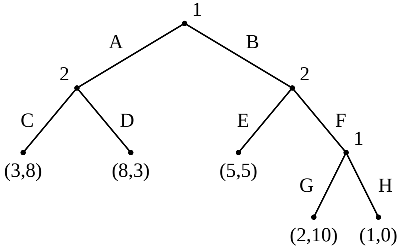


	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

- What are the (three) pure-strategy equilibria?

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form

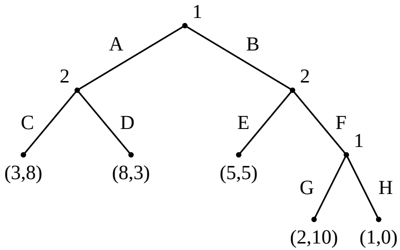


	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

- What are the (three) pure-strategy equilibria?
  - $(A,G),(C,F)$
  - $(A,H),(C,F)$
  - $(B,H),(C,E)$

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form



	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

- What are the (three) pure-strategy equilibria?
  - $(A,G),(C,F)$
  - $(A,H),(C,F)$
  - $(B,H),(C,E)$

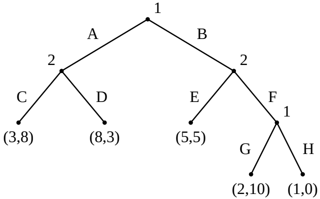
Lecture Overview

Recap

Subgame Perfection

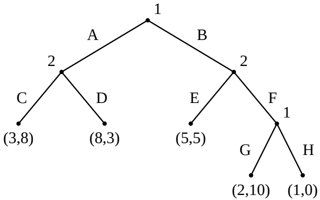
Backward Induction

Subgame Perfection



- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him

Subgame Perfection



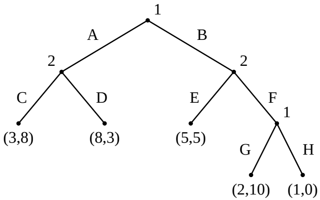
- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him
  - He does it to threaten player 2, to prevent him from choosing F, and so gets 5
    - However, this seems like a non-credible threat
    - If player 1 reached his second decision node, would he really follow through and play H?

Formal Definition

- Define subgame of G rooted at h:
  - the restriction of G to the descendents of H.
- Define set of subgames of G:
  - subgames of G rooted at nodes in G

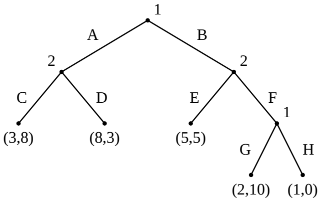
- s is a subgame perfect equilibrium of G iff for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'
- Notes:
  - since G is its own subgame, every SPE is a NE.
  - this definition rules out "non-credible threats"

Back to the Example



- Which equilibria from the example are subgame perfect?

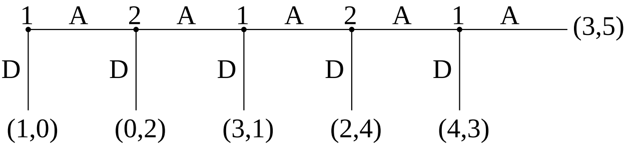
Back to the Example



- Which equilibria from the example are subgame perfect?
  - $(A, G), (C, F)$ is subgame perfect
  - (B, H) is an non-credible threat, so $(B, H), (C, E)$ is not subgame perfect
  - (A, H) is also non-credible, even though H is "off-path"

Lecture Overview

Centipede Game



- Play this as a fun game...