Homework #2

ISCI 330 Game Theory

NOTE: You don't need to expand fractions into decimals. Be sure to show your work. **(With corrections to problem 2j)**

1) **(13 Points)** You are biologist studying the symbiotic relationship between a newly discovered species of cleaner fish and the host species that they clean. In each interaction a cleaner fish can cooperate (C) by properly cleaning its host or defect (D) by taking little bites out of its host's mucosal membrane while cleaning its mouth. The host fish can also cooperate (C) by holding still and keeping its mouth wide open while it is being cleaned, or defect (D) by trying to close its mouth on the cleaner fish and eat it. Your graduate student has been collecting utility data on these interactions. (Note that as a biologist you might call these "fitness" values instead of "utility.") Unfortunately your student got a fancy job before finishing the project, and left before all the utility information was complete. The **x** and the **y** below indicate the utility numbers that are missing.

Host fish

Cleaner fish		**C**	**D**
	C	4, 6	5**, x**
	D	**y,** 36	8, 6

a) **(6 pts)** After countless hours of observations (scuba diving off the coast of Fiji) you now know that on average cleaner fish cooperate in 5/6 of their interactions when they meet a host fish, while host fish cooperate in 3/4 of these interactions. You hypothesize that these probabilities represent a mixed strategy Nash equilibrium. Given your hypothesis and the known utility values above, calculate the values of x and y. (Be sure to show your work, on the back if necessary.)

b) **(4 pts)** Given the completed payoff matrix, what is the expected utility in an encounter for cleaner fish? And for host fish?

c) **(3 pts)** Suppose cleaner fish in their observations of host fish figured out that the host fish cooperate with probability ¾ (the same proportion you observed). Could this information be useful to the cleaner fish? Explain your answer.

2) **(62 Points)** In the soccer penalty kick situation discussed in class, kickers tend to be better at kicking to one side or the other. On the other hand, goalies tend to block equally well to either side. We'll model this scenario as a game with the following actions for each player: kick left (L) or right (R) for the kicker, dive left (L) or right (R) for the goalie.[1]

Kicker A and goalie B are about to have a showdown at the end of a championship game. Kicker A knows that she is better at kicking to her left. In cases where the goalie doesn't block the kick, she misses the goal on average 15% of the time when she kicks to the left, and misses 30% of the time when she kicks to the right. Goalie B's past performance reveals that when she dives to the same side as a kick she is able to block a goal 80% of the time. When she dives to the opposite side as a kick, she never blocks the goal.

a) **(8 pts)** First let's figure out the probability of a goal in each of the four combinations of kicker A and goalie B actions.

i) Given that kicker A kicks left (L) and goalie B dives to the same side (L), what is the probability a goal is scored?

ii) Given that kicker A kicks right (R) and goalie B dives to the same side (R), what is the probability a goal is scored?

iii) Given that kicker A kicks left (L) and goalie B dives to the opposite side (R), what is the probability a goal is scored?

iv) Given that kicker A kicks right (R) and goalie B dives to the opposite side (L), what is the probability a goal is scored?

b) **(8 pts)** The utility for kicker A when she makes a goal is 100 and when does not make a goal her utility is 0. The utility for goalie B when a goal is scored is 0 and when a goal is not scored is 100. Given this information and your answers above, fill in the following utility payoff matrix for this upcoming penalty kick.

Goalie B **Show your work here:**

		Left (L)	**Right (R)**
Kicker A	**Left (L)**	___,___	___,___
	Right (R)	___,___	___,___

[1] That is, when the goalie dives left and the kicker kicks left, they've both gone to the same side of the net.

c) **(6 pts)** Each player is rational and wants to maximize her expected utility based on the game matrix in b). Kicker A's team hires you as their game theorist and you can communicate with kicker A via hidden wireless headphones under her toque. You use a random number generator and the rational (mixed Nash equilibrium) probability distribution to tell kicker A which way to kick, left or right, in the upcoming penalty kick. Calculate the probability you tell her to kick left. Assuming that goalie B is also getting rational advice, calculate the probability that the goalie dives to the left (kicker's left) side. (Show your work – on the back if necessary.) Be sure your final answers are clearly labelled.

d) **(4 pts)** Given the mixed strategies identified in c), what is the expected utility for the kicker? And for the goalie?

e) **(3 pts)** Suppose instead of both players being rational, the kicker is superstitious and always kicks to the right on her birthday regardless of what you tell her, and the game is being played on her birthday. The game theorist for goalie B's team finds out this information and knows the kicker will kick right. Does this change the advice the other team's game theorist should give goalie B? Explain your answer.

f) **(3 pts)** Kicker A is having a bad season and is in danger of being fired if she fails to score on this kick. Thus, she wants to maximize her worst-case expected utility. Should you advise her any differently? If so, what (mixed? pure?) strategy do you advise her to play? (show your work, on the back if necessary)

g) **(12 pts)** Now consider a case where goalie B is a “glory hound” and has a higher utility for actually blocking a kick herself than for when the kicker just misses. Blocking a kick still gives her a utility of 100 and a goal being scored still gives her a utility of 0, but the kicker missing without her touching the ball only gives her a utility of 50. Given this new information and your answers in part a), fill in the following utility payoff matrix for this upcoming penalty kick.

Goalie B **Show your work here (and/or on the back):**

		Left (L)	**Right (R)**
Kicker A	**Left (L)**	___,___	___,___
	Right (R)	___,___	___,___

h) **(6 pts)** What is the Nash equilibrium of this new game?

i) **(4 pts)** Given these mixed strategies in h), what is the expected utility for the kicker? And for the goalie?

j) **(4 pts) [corrections underlined]** The center on **goalie B's** team watching the penalty kick has the same utility function that you calculated for the **goalie** in b). How much is the center's expected utility diminished by **goalie B's** new "glory hound" Nash equilibrium strategy (compared to if she wasn't a glory hound and played her Nash equilibrium strategy calculated in c)?

k) **(4 pts)** Reconsider the question from part (f) given the new game from part (g). Should you advice the kicker differently in this new game. If so, what (mixed? pure?) strategy would you advise kicker A to play now? (show your work, on the back if necessary)

3) **(10 Points)** Find the Nash equilibrium of the following game. (Sorry, but we can't come up with a fun story for this problem. If you're feeling creative, though, supply your own! We'll give one bonus point for the best story we receive. ☺)

16, 7	8, 6	10, 10
11, 6	15, 2	9, 14
6, 12	7, 2	10, 8
2, 8	14, 15	13, 2

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. Getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

List any non-course materials you referred to:

Signature:

Fill in this page and include it with your assignment submission.