Perfect-Information Extensive Form Games

Week 4

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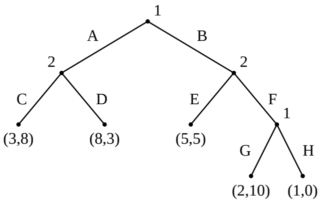
- Play the ultimatum game, splitting 100 dollars.
- How many pure strategies does each player have in this game?
- Define a pure strategy in a PIEF game.
- What is the induced normal form?
- Define a Nash equilibrium in a PIEF game.

Subgame Perfection



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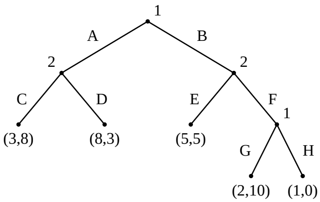

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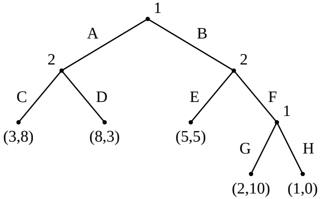

- What's intuitively wrong with the equilibrium $(B, H), (C, E)$?
- What's a subgame-perfect equilibrium?
- What are the subgame-perfect equilibria of this game?

Computing Subgame Perfect Equilibria

Idea: Identify the equilibria in the bottom-most trees, and adopt these as one moves up the tree

```
function BackwardInduction (node h) returns u(h)
if h ∈ Z then
    return u(h)
best_util ← −∞
forall a ∈ χ(h) do
    util_at_child ← BackwardInduction(σ(h, a))
    if util_at_child_ρ(h) > best_util_ρ(h) then
        best_util ← util_at_child
return best_util
```

- $util\_at\_child$ is a vector denoting the utility for each player
- the procedure doesn't return an equilibrium strategy, but rather labels each node with a vector of real numbers.
  - This labeling can be seen as an extension of the game's utility function to the non-terminal nodes
  - The equilibrium strategies: take the best action at each node.

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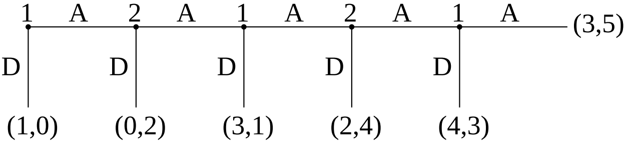
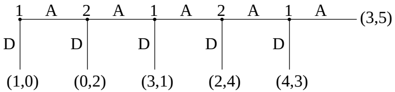
- For zero-sum games, BackwardInduction has another name: the minimax algorithm.
  - Here it's enough to store one number per node.
  - It's possible to speed things up by pruning nodes that will never be reached in play: "alpha-beta pruning".

Centipede Game



- Play this as a fun game...

Backward Induction



- What happens when we use this procedure on Centipede?
  - In the only equilibrium, player 1 goes down in the first move.
  - However, this outcome is Pareto-dominated by all but one other outcome.
- Two considerations:
  - practical: human subjects don't go down right away
  - theoretical: what should you do as player 2 if player 1 doesn't go down?
    - SPE analysis says to go down. However, that same analysis says that P1 would already have gone down. How do you update your beliefs upon observation of a measure zero event?
    - but if player 1 knows that you'll do something else, it is rational for him not to go down anymore... a paradox
    - there's a whole literature on this question