Game Theoretic Analysis of Random Access in Wireless Networks

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Abstract

In the past three decades, random access mechanism, such as slotted ALOHA, has been widely adopted as an efficient protocol to coordinate the medium access among competing users. The early works on random access mechanisms focus on the analysis of their performance from the system perspective, where mobile users are considered as homogeneous devices. In recent years, game theoretic approaches have been applied to study random access mechanisms from the user perspective, where users are assumed to be intelligent who make transmission decisions to maximize their own utilities. The game theoretic analysis can provide significant insights for designing random access protocols of next generation wireless systems with autonomous and heterogeneous mobile users. This paper studies the current game theoretic approaches for conventional random access problem and further extend the analysis to a new random access scenario where users can choose different transmission mode when making a channel access decision.

Keywords: Random access, slotted ALOHA, game theory.

I. INTRODUCTION

Wireless channel is shared in nature. When several users attempt to access the same channel simultaneously, their transmissions may fail due to collisions. In the past three decades, random access mechanisms, such as ALOHA [1], Carrier Sense Multiple Access (CSMA) [2], and their corresponding variations have been widely studied as efficient methods to coordinate the medium access among competing users. In these mechanisms, each user either maintains a persistent transmission probability or adjusts a backoff window to resolve contention. For instance, in slotted ALOHA mechanism, a user transmits a packet with certain probability during each time slot to reduce contention. In CSMA mechanism, a user maintains a backoff window and waits for a random amount of time bounded by the backoff window before a transmission (or retransmission). Performance of these random access mechanisms have been studied extensively from the system perspective, where mobile users are considered as homogeneous devices that always follow the transmission protocol. In recent years, with the popularity of open source operating systems, mobile devices, such as smart phones and tablet PCs, have been transformed into intelligent terminals. Wireless users can easily modify the transmission protocol of their devices to maximize their own benefits. In consequence, designing fair and efficient random access mechanisms for systems with heterogeneous and intelligent users becomes more challenging.

Recent papers start to look at the random access problem from the user perspective, where users are considered as intelligent and rational individuals who make decisions to maximize their own benefits [3] [4] [5] [6] [7]. In these papers, game theory has been applied to modeling and analyzing the random access process with autonomous and heterogeneous mobile users. As a powerful tool to study the interactions among intelligent and rational individuals, game theory has the potential to provide insights and analytical approach to the design of efficient random access mechanisms. In general, the game theoretic approach for analyzing random access contains three steps: game formulation, equilibrium analysis, and mechanism design. The first step is to model the random access process as a game. Typically, the mobile users are considered as players, and their transmission decisions are the actions. Each user is also assigned a utility function, which characterizes the user's satisfaction of a particular outcome (i.e., the utility can be the gain from a successful access minus the transmission cost). Under different assumptions on the users' decision making process (i.e., private objective and available information of other users), the random access game can be formulated as a single shot game with deterministic strategy [4], or a game with probabilistic strategy [3], or a Bayesian game [5]. Once the random access game is formulated, the next step is to analyze the equilibria of the game. Specifically, the existence of Nash equilibrium and corresponding conditions are analyzed in this step. The equilibrium analysis provides significant insights on whether the system can be operated at a stable state such that users do not change their decisions unilaterally. Finally, based on the equilibrium analysis, an efficient random access mechanism can be designed to guide users to operate at a desired equilibrium state.

This paper studies game theoretic approaches for random access in wireless networks. A time slotted system is considered, where time is divided into slots, and users make their decisions to access a shared channel at the beginning of each time slot. The proposed study of random access in this paper is two-fold. First, two existing game theoretic approaches with different game formulations are reviewed and compared. Second, based on these game theoretic approaches, a new random access scenario is studied, where users can choose a transmission mode (from two available modes) when making the access decisions. A new game formulation is provided and the corresponding equilibria are analyzed. This work may provide insights for designing random access mechanisms for next generation wireless systems.

The rest of this paper is organized as follows. Section II describes a general system model. In Section III, two game theoretic approaches are studied and compared. Section IV provides a game theoretic analysis for random access with transmission mode selection. Section V concludes the paper.

II. SYSTEM MODEL

A general random access system is considered in this paper, where a number of $N > 1$ wireless users are contending to access a common channel for their own transmissions. For example, in the cellular system, several mobile users share the same channel to request the uplink communication from a base station. Time is slotted, and each transmission occupies one time slot. All the users are synchronized so that each transmission starts at the beginning of a slot.

To characterize the transmission collision, the conventional collision model is used, where a transmission is successful when there is no other ongoing transmission. It implies that during each time slot only one user can access the channel successfully. This simplified channel model is widely used for performance analysis, and it is suitable for the scenario where the received signal strength from different users are comparable at the base station. In Section IV, a multi-antenna scenario is considered where multiple data streams can be transmitted successfully simultaneously, which will be discussed later.

To simplify the analysis, it is also assumed that users always have data to transmit during each time slot, and they will be notified through feedback once the transmission is failed.

III. GAME THEORETIC ANALYSIS OF RANDOM ACCESS

This section studies recent game theoretic approaches on analyzing random access in the literature. Two different non-cooperative game formulations, i.e., game with deterministic strategy and game with probabilistic strategy are discussed. Specifically, the game formulation and corresponding equilibrium analysis are provided, and comments on these game theoretic approaches are given.

A. *Random Access Game with Deterministic Strategy*

For the slotted system considered in Section II, users may make decisions on whether to transmit or not at the beginning of each time slot. Consider the user interactions during one time slot, the random access process can be formulated as a non-cooperative game with deterministic strategies for each user [4]. In this game formulation, users are the players, and each of them have two deterministic strategies–transmit and backoff. The conventional collision model is assumed, where a transmission fails when there is more than one transmissions. A user i gains a utility of 1 if a transmission is successful (for simplicity, the utility is normalized to one unit) and costs c_i when a transmission fails. A formal definition of this game formulation is as follows.

Definition 1: The n-player one-shot random access game with deterministic strategies is the game $G(n, \boldsymbol{c}) = \langle \mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$ such that $\mathcal{N} = \{1, 2, \ldots, n\}$, $\mathcal{A}_i = \{0, 1\}$ for all $i \in \mathcal{N}$, where 1 means transmission and 0 means backoff, $\boldsymbol{c} = (c_i)_{i\in\mathcal{N}}$ where c_i is the cost of an unsuccessful transmission for user i, and the utility function u_i for all $i \in \mathcal{N}$ is defined as

$$u_i(\mathbf{a}) = \begin{cases} 0 & \text{if } a_i = 0, \\ 1 & \text{if } \|\mathbf{a}\|_{l^1} = 1 \text{ and } a_i = 1, \\ -c_i & \text{if } \|\mathbf{a}\|_{l^1} \geq 2 \text{ and } a_i = 1, \end{cases} \tag{1}$$

where a_i is user i's action and $\|\cdot\|_{l^1}$ is the l^1 norm for the vectors in $\mathbb{R}^n$, which is defined as the sum of the absolute values of the components of a vector.

The equilibria analysis of this game begins with a two-user case ($G(2, \boldsymbol{c})$) as shown in Fig. 1. For the two-player game $G(2, \boldsymbol{c})$, it is easy to prove that there are two pure strategy Nash equilibria and one fully mixed strategy equilibrium. At the pure strategy equilibria, one of the users always transmits and the other chooses to backoff. At the mixed strategy equilibrium, both users randomize their strategies between transmit and backoff with transmission probabilities $\Pr\{a_1 = 1\} = 1/(c_2+1)$ and $\Pr\{a_2 = 1\} = 1/(c_1 + 1)$, respectively. This result implies that given a transmission probability assignment $\mathbf{p} = (p_1, p_2) \in (0, 1)^2 \bigcup\{(1, 0), (0, 1)\}$, there exists a cost pair (c_1, c_2) such that users' transmission probabilities coincide with $\mathbf{p}$ at one of the Nash equilibria. The equilibria analysis can be further extended to the n-user scenario, and the following theorem is provided [4].

Theorem 1:

Part I: Let $\pi(\mathbf{c}) = \prod_i (c_i/(1 + c_i))$, and $c_{\max} = \max_{i\in\mathcal{N}} c_i$. If the users satisfy the regularity condition

$$\left(\frac{c_{\max}}{1 + c_{\max}}\right)^{\frac{n}{n-1}} \leq \frac{c_i}{1 + c_i} \tag{2}$$

for all $i \in \mathcal{N}$, then $G(n, \mathbf{c})$ has $2^n - 1$ Nash equilibria such that

(i) n of them are in pure strategies.

(ii) One of them is an fully mixed equilibrium, and it is given by $\mathbf{p}^* = (p_i^*)_{i\in\mathcal{N}}$ such that $p_i^* = 1 - ((1 + c_i)/c_i)(\pi(\mathbf{c}))^{1/(n-1)}$ for all $i \in \mathcal{N}$.

(iii) For the rest of them, there is a group of players $\mathcal{N}_0$ randomizing between transmit and backoff according to a nondegenerate probability distribution, and the rest of the players choose ”always backoff” strategy. Moreover, if $\mathbf{c}' = (c_i)_{i\in\mathcal{N}_0}$, then transmission probabilities for these users are given by $p_i^* = 1 - ((1+c_i)/c_i)\pi(\mathbf{c}'))^{1/(|\mathcal{N}_0|-1)}$ for all $i \in \mathcal{N}_0$.

Part II: Let $\mathbf{c} \in \mathbb{R}^n$ and $\mathcal{N}_0 \subseteq \mathcal{N}$ with $2 \leq |\mathcal{N}_0| \leq n$. Then, $G(n, \mathbf{c})$ has n pure-strategy Nash equilibria. Moreover, any mixed-strategy profile $\mathbf{p}^* = (p_i^*)_{i\in\mathcal{N}}$ such that users in $\mathcal{N}_0$ mix between transmit and backoff actions according to a nondegenerate probability distribution, and users in $\mathcal{N} - \mathcal{N}_0$ backoff with probability 1 is a Nash equilibrium if and only if

$$p_i^* = 1 - \left(\frac{1+c_i}{c_i}\right)(\pi(\mathbf{c}'))^{1/(|\mathcal{N}_0|-1)}$$

for $i \in \mathcal{N}_0$, and $c_i/(1+c_i) > \pi(\mathbf{c}')^{1/(|\mathcal{N}_0|-1)}$ for all $i \in \mathcal{N}$ (with $\geq$ if $i \in \mathcal{N} - \mathcal{N}_0$), where $\mathbf{c}' = (c_i)_{i\in\mathcal{N}_0}$.

Theorem 1 provides an explicit characterization of the Nash equilibria for the n-user random access game, where either all users use pure strategy, or fully mixed strategy, or some of them use mixed strategy while others backoff. Moreover, it provides the necessary and sufficient conditions for a particular transmission probability profile to be a Nash equilibrium. Similar to the two-user game, for a given transmission probability profile (except the rear ones such as $\mathbf{p} = \mathbf{0}$ and $\mathbf{p} = (p_i)_{i\in\mathcal{N}}$ where there exists $i, j \in \mathcal{N}$ with $p_i = 1$ and $p_j > 0$), there exists a cost function profile such that users’ transmission probabilities coincide with a Nash equilibrium. The equilibria analysis implies that any desired transmission probability assignment for the users can be realized in an equilibrium, by manipulating the costs of users. For example, the base station can charge the users for channel access such that the perceived costs of failed transmissions at the usres satisfy the corresponding equilibrium condition. This result provides valuable insights in designing random access mechanisms in order for the system to operate at a desired stable state.

B. Random Access Game with Probabilistic Strategy

Instead of modeling a user’s decision as a deterministic strategy, some papers use probabilistic strategy when formulating the game [3]. The motivation of this formulation is to reverse engineer the current random access protocols (such as slotted ALOHA) and to provide local operation rules to achieve system-wide performance objectives. In a general formulation of random access game with probabilistic strategy, each user i is associated with a transmission probability p_i and a contention measure q_i. A user's strategy is to choose a persistent transmission probability p_i (which may be updated after a certain period). The utility of user i is a function of p_i and q_i, which is defined as the difference between the expected gain of a successful transmission and the expected cost of contention. A formal definition of this formulation is as follows [3].

Definition 2: The n-player random access game with probabilistic strategies is the game $\mathcal{G} = \langle \mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$, where $\mathcal{N} = \{1, 2, \ldots, n\}$, $\mathcal{A}_i = \{p_i | p_i \in [\mu_i, \omega_i]\}$ with $0 \leq \mu_i < \omega_i < 1$, and the utility function $u_i(\mathbf{p}) = U_i(p_i) - p_i q_i(\mathbf{p})$ with expected gain $U_i(p_i)$ and contention measure $q_i(\mathbf{p})$.

The above formulation characterizes a long term random access process, and the strategy of a user is constrained to be strictly less than 1 in order to prevent a user from exclusively occupying the channel. The analysis for this game mainly focus on two issues: whether a Nash equilibrium exists and how to design a local operation mechanism to achieve the equilibrium.

For the game defined above, the equilibrium properties are summarized in the following theorems [].

Theorem 2: The random access game $\mathcal{G}$ with probabilistic strategy has a unique Nash equilibrium if the following conditions hold:
(a) The expected gain function $U_i(\cdot)$ is continuously differentiable, strictly concave, and with finite curvatures that are bounded away from zero, i.e., there exist some constants μ and λ such that $1/\mu \geq -1/U_i''(p_i) \geq 1/\lambda > 0$.
(b) Let $\gamma(\mathbf{p}) = \prod_{i\in\mathcal{N}}(1 - p_i)$ and denote the smallest eigenvalue of $\nabla^2\gamma(\mathbf{p})$ over $\mathbf{p}$ by $\nu_{\min}$. Then $-\mu - \nu_{\min} < 0$.
Furthermore, denote the equilibrium action profile as $\mathbf{p}^*$, then, $U_i'(p_i^*) = q_i(\mathbf{p}^*)$

Theorem 3: Define $\Gamma_i(p_i) = (1 - p_i)(1 - U_i'(p_i))$ $\forall i \in \mathcal{N}$. If the random access game $\mathcal{G}$ has a nontrivial Nash equilibrium, it must be unique if $\Gamma_i(p_i)$ for all $i \in \mathcal{N}$ are all strictly increasing or all strictly decreasing.

The proofs of the above theorems are provided in [3] and are omitted here for brevity. It is shown that under certain conditions, the random access game has a unique nontrivial Nash equilibrium, which can be found through the specification of expected gain function $U_i(p_i)$ and the contention measure q_i. For example, under the conventional collision model, the contention measure can be chosen as the conditional collision probability, where

$$q_i(\mathbf{p}) = 1 - \prod_{j\in\mathcal{N}, j\neq i} (1 - p_i),\ \ i \in \mathcal{N}.$$

Then, given a set of $U_i(p_i)$, the nontrivial Nash equilibrium can be found numerically according to $U_i'(p_i^*) = q_i(\mathbf{p}^*)$ for all $i \in \mathcal{N}$. If the gain functions are the same, then the game is said to have homogeneous users. Theorem 2 and Theorem 3 imply that if there exists a nontrivial Nash equilibrium for the game with homogeneous users, it must be unique and symmetric, where $p_i^* = p_j^*$ for all $i, j \in \mathcal{N}$.

The next question is how to design a local operation rule to achieve the equilibrium. In a purely distributed random access scenario, a user may not know the others' gain functions, which makes it difficult to find the Nash equilibrium individually. The random access process may not operate at a stable state at the beginning and may evolve dynamically during the play. In this case, it is necessary to design an operation mechanism for the users so that they can update their transmission probability periodically. Typically, a selfish user prefers to choose the ”best response” action according to his/her contention measure. That is, a user may update the transmission probability according to

$$p_i(t+1) = \arg\max_{p\in\mathcal{A}_i} (U_i(p) - pq_i(\mathbf{p}(t))),$$

where $\mathbf{p}(t)$ is the contention measure during period t, and $p_i(t+1)$ is the transmission probability for period $t+1$. If this update mechanism reaches a steady state, then this state is a Nash equilibrium. However, the random access game may not have dominant strategy equilibrium, and therefore, such steady state may never be reached. An alternative strategy update mechanism named the gradient play is proposed in [3]. In this mechanism, each user adjusts the transmission probability gradually in a gradient direction based on the observations of other users' actions. Mathematically, user $i \in \mathcal{N}$ updates his/her strategy according to

$$p_i(t+1) = [p_i(t) + f_i(p_i(t))(U_i(p_i(t)) - q_i(\mathbf{p}(t)))]^{\mathcal{A}_i},$$

where $f_i(\cdot) > 0$ is the step size function, and $\mathcal{A}_i$ denotes the projection onto the user i's strategy space. It is shown in [3] that this gradient play mechanism converges to the unique Nash equilibrium under certain conditions, as stated in the following theorem.

Theorem 4: Suppose the assumptions in Theorem 2 hold. The gradient play mechanism converges to the unique Nash equilibrium of random access game $\mathcal{G}$ if, for any $i \in \mathcal{N}$, the step size $f_i < \frac{2}{\lambda+n-1}$. Although the gradient play mechanism does not provides the best response during a period, it guarantees that the Nash equilibrium can be achieved by selecting the step size properly. In practice, the system designer can design a utility function and adopt the gradient play mechanism for the users in order to achieve system-wide performance objectives, such as fairness and throughput. Examples of such mechanism design can be found in [], and is omitted here for brevity.

C. Further Comments

So far, two game theoretic approaches for analyzing random access have been discussed. Although both of these approaches adopt non-cooperative game theory, there are several fundamental differences between the analysis. First, the former game theoretic approach conduct a bottom-up analysis. The game is formulated considering users' various selfish behaviors. On the contrary, the latter approach can be viewed as a top-down method, which reverse engineer the existing random access protocols. The game formulation neglects users' certain selfish behaviors, i.e., the pure strategy actions. Second, the objective of the former approach is to analyze users' behaviors and the equilibrium properties of the game in order to provide guidelines on designing random access mechanisms. The latter approach mainly focus on developing local operation rules (i.e. the update mechanism) to achieve a equilibrium state. Moreover, the users' utility function of the latter approach may be manipulated by the system designer to achieve a system-wide objectives such as fairness and throughput, without considering the incentive issues. Therefore, when analyzing random access problems, these two approaches should be applied carefully considering the design objectives.

IV. Random Access with Transmission Mode Selection

In this section, the aforementioned bottom-up game theoretic approach is applied to analyzing a new random access scenario, where multiple data streams can be transmitted simultaneously without collision. In recent wireless systems, i.e., the LTE-Advanced system [8], multi-antenna technique has been employed by both the base station and the mobile users. With multiple antennas, several data streams can be simultaneously transmitted from a user to the base station, and the base station is able to decode multiple data streams from different users. This amazing technique brings new challenge for designing uplink random access mechanisms: in addition to decide whether to transmit during a time slot, a user should also select a transmission mode (on how many data streams to transmit) to achieve certain performance objective. The behavior of selfish users in such scenario has not been studied, which motivates the work in this section.

Consider a wireless system as described in Section II, with additional assumptions that all users and the base station are equipped with two omnidirectional antennas. Then, each user has two transmission mode : transmit one data stream and transmit two data streams. The base station is capable of receiving two data streams simultaneously, i.e., one data stream from two users or two data streams from a single user. A collision happens when more than two data streams are transmitted simultaneously. The aforementioned random access process can be modeled as a non-cooperative game with deterministic strategies as follows.

Definition 3: The n-player random access game with transmission mode selection is the game $\mathcal{G}$ = $\langle \mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$ such that $\mathcal{N} = \{1, 2, \ldots, n\}$, and $\mathcal{A}_i = \{0, 1, 2\}$, where 0 means backoff, 1 means transmit one data streams and 2 means transmit two data streams. The utility function u_i for all $i \in \mathcal{N}$ is defined as

$$u_i(\mathbf{a}) = \begin{cases} 0 & \text{if } a_i = 0, \\ g_{i1} & \text{if } \|\mathbf{a}\|_{l^1} \leq 2 \text{ and } a_i = 1, \\ g_{i2} & \text{if } \|\mathbf{a}\|_{l^1} = 2 \text{ and } a_i = 2, \\ -c_{i1} & \text{if } \|\mathbf{a}\|_{l^1} > 2 \text{ and } a_i = 1, \\ -c_{i2} & \text{if } \|\mathbf{a}\|_{l^1} > 2 \text{ and } a_i = 2, \end{cases} \tag{3}$$

where g_{ik} is the gain of user i when using transmission mode k with no collision, c_{ik} is the cost of user i when using transmission mode k with collision. a_i is user i's action and $\|\cdot\|_{l^1}$ is the l^1 norm for the vectors in $\mathbb{R}^n$, which is defined as the sum of the absolute values of the components of a vector.

To analyze the equilibrium of the n-player game, it is better to begin with the simplist case with two players. A normal form expression of the two-layer random access game is shown in Fig. 2. From Fig. 2, it is easy to verify that there exist three pure strategy Nash equilibria, with the strategy profile $(2,0)$, $(1,1)$, $(0,2)$, respectively. There may also exist a mixed strategy Nash equilibrium. Denote user i's transmission probability with mode k as $p_{ik}, k \in \mathcal{A}_i$. Then, at the mixed strategy equilibrium, user 1's strategy should make user 2 indifferent among all the choices. That is,

$$0 = p_{11}g_{21} - p_{12}c_{21} + (1 - p_{11} - p_{12})g_{21} = -p_{11}c_{22} - p_{12}c_{22} + (1 - p_{11} - p_{12})g_{22}. \tag{4}$$

Solving (4) gives that $p_{11} = \frac{g_{22}c_{21} - g_{21}c_{22}}{(g_{22}+c_{22})(g_{21}+c_{21})}$ and $p_{12} = \frac{g_{21}}{g_{21}+c_{21}}$. Similarly, the mixed strategy of user 2 at the Nash equilibrium can be derived as $p_{21} = \frac{g_{12}c_{11} - g_{11}c_{12}}{(g_{12}+c_{12})(g_{11}+c_{11})}$ and $p_{22} = \frac{g_{11}}{g_{11}+c_{11}}$. Note that this fully mixed Nash equilibrium exists if and only if $g_{12}c_{11} - g_{11}c_{12} > 0$ and $g_{22}c_{21} - g_{21}c_{22} > 0$. Since user i achieves zero utility by taking action $a_i = 0$, the expected utility for both users at the mixed strategy Nash equilibrium are zero.

In summary, there are three pure strategy equilibria and one mixed strategy equilibrium for the two-player game. The next question is which equilibrium is preferred by the users and how users behave in practice. Clearly, a selfish user prefers an equilibrium where s/he gains the maximum utility. In this game, the utilities associated with the three pure strategy Nash equilibria are $(g_{12}, 0)$, (g_{11}, g_{11}), and $(0, g_{22})$, and the mixed strategy Nash equilibrium gives a utility $(0,0)$. It is obvious that user 1 prefers $(g_{12}, 0)$ and user 2 prefers $(0, g_{22})$ since they give the highest utility for each user, respectively. However, user 1 may realize that the action 0 may never be chosen by user 2 since user 2 is selfish. As a result, the equilibrium point $(g_{12}, 0)$ may not be realized and taking action $a_i = 2$ is highly risky. The same argument applies to user 2. Finally, the users may choose to play at the Nash equilibrium point $(1,1)$, where both of them are happy with utility (g_{11}, g_{21}). This implies that in a two user random access process with transmission mode selection, users may coordinate with each other naturally. This observation provides valuable insights in designing incentive-compatible random access mechanisms.

The next step is to extend the analysis to n-player game. To simplify the analysis, users are assumed to be homogeneous, where $g_{ik} = g_{jk} = g_k, c_{ik} = c_{jk} = c_k, \forall\ i, j \in \mathcal{N}, k \in \{0,1,2\}$. Based on the equilibria analysis of the two-player game and Theorem 1 in Section III, it can be shown that the n-player game has three types of Nash equilibria, which have similar structure to those in Theorem 1.

(i) First, it is easy to verify that there are $n + C_n^2 = \frac{1}{2}n(n+1)$ pure strategy Nash equilibria, which can be divided into two classes. In the first class, only one of the users transmit with two data streams and others backoff. In the second class, any two of the users transmit one data stream each and others backoff.

(ii) Second, there may exist a fully mixed strategy Nash equilibrium. The same approach as that in the two-player game can be applied to find it. Since users are homogeneous, there may exists a symmetric equilibrium. Denote the corresponding probability of transmit one data stream at any user as p_1 and the probability of backoff as p_0, respectively. Then any user i should feel indifferent among all the choices at the mixed strategy equilibrium, which leads to

$$0 = g_1(C_{n-1}^1 p_1 p_0^{n-2} + p_0^{n-1}) - c_1[1 - (C_{n-1}^1 p_1 p_0^{n-2} + p_0^{n-1})] = g_2 p_0^{n-1} - c_2(1 - p_0^{n-1}). \tag{5}$$

Solving (5) gives $p_0 = \left(\frac{c_2}{g_2+c_2}\right)^{\frac{1}{n-1}}$ and $p_1 = \frac{g_2c_1 - g_1c_2}{(g_1+c_1)(n-1)c_2} \cdot \left(\frac{c_2}{g_2+c_2}\right)^{\frac{1}{n-1}}$. Obviously, this fully mixed Nash equilibrium exists when $g_2c_1 - g_1c_2 > 0$.

(iii) Finally, at the rest Nash equilibria, some of the users randomize their choices while others all choose to backoff. Specifically, if $g_2c_1 - g_1c_2 > 0$, for a subset of users $\mathcal{N}'$, where $|\mathcal{N}'| = n' > 2$, there exists a Nash equilibrium such that any user $i \in \mathcal{N}'$ randomizes his/her choices with $p_0 = \left(\frac{c_2}{g_2+c_2}\right)^{\frac{1}{n'-1}}$ and $p_1 = \frac{g_2c_1 - g_1c_2}{(g_1+c_1)(n'-1)c_2} \cdot \left(\frac{c_2}{g_2+c_2}\right)^{\frac{1}{n'-1}}$, while all other users choose to backoff. This result follows Theorem 1 and the proof is omitted here for brevity.

Unlike the two-player game, the pure strategy equilibria and partially mixed equilibria in the n-player game only benefit some users, which are not fair in practice. Therefore, these Nash equilibria are neither preferred by all the users nor preferred by the system designer. Users may randomize their choices according to the fully mixed Nash equilibrium to gain access of the channel.

V. CONCLUSION

In this paper, game theoretic analysis for random access in wireless networks was studied. Two game theoretic approaches, a bottom-up approach and a top-down approach were discussed under different game formulations, and their difference were compared. Further more, the bottom-up approach was applied to a new random access scenario with transmission mode selection at each user. A new game was formulated and the corresponding equilibrium was analyzed. This work may provide valuable insights in designing random access mechanisms for next generation wireless systems.

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Player 1	Player 2		
		1	0
	1	$-c_1, \ -c_2$	1,0
	0	0,1	0,0

Fig. 1. The two-user game with deterministic strategies.

	Player 2			
Player 1		0	1	2
	0	0,0	0,g_{21}	0, g_{22}
	1	g_{11},0	g_{11}, g_{21}	-c_{11},- c_{22}
	2	g_{12},0	-c_{12}, -c_{21}	-c_{12},- c_{22}

Fig. 2. The two-user game with transmission mode selection.