Auction Theory I

Lecture 18

Lecture Overview

Motivation

- Auctions are any mechanisms for allocating resources among self-interested agents
- Very widely used
  - government sale of resources
  - privatization
  - stock market
  - request for quote
  - FCC spectrum
  - real estate sales
  - eBay

CS Motivation

- resource allocation is a fundamental problem in CS
- increasing importance of studying distributed systems with heterogeneous agents
- markets for:
  - computational resources (JINI, etc.)
  - P2P systems
  - network bandwidth
- currency needn't be real money, just something scarce
  - that said, real money trading agents are also an important motivation

Lecture Overview

Some Canonical Auctions

- English
- Japanese
- Dutch
- First-Price
- Second-Price
- All-Pay

English Auction

English Auction

- auctioneer starts the bidding at some "reservation price"
- bidders then shout out ascending prices
- once bidders stop shouting, the high bidder gets the good at that price

Japanese Auction

Japanese Auction

- Same as an English auction except that the auctioneer calls out the prices
- all bidders start out standing
- when the price reaches a level that a bidder is not willing to pay, that bidder sits down
  - once a bidder sits down, they can't get back up
- the last person standing gets the good

- analytically more tractable than English because jump bidding can't occur
  - consider the branching factor of the extensive form game...

Dutch Auction

Dutch Auction

- the auctioneer starts a clock at some high value; it descends
- at some point, a bidder shouts "mine!" and gets the good at the price shown on the clock

First-, Second-Price Auctions

First-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount of his bid

Second-Price Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount bid by the second-highest bidder

All-Pay auction

All-Pay Auction

- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- everyone pays the amount of their bid regardless of whether or not they win

Auctions as Structured Negotiations

Any negotiation mechanism that is:

- market-based (determines an exchange in terms of currency)
- mediated (auctioneer)
- well-specified (follows rules)

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
  - who can bid, when
  - what is the form of a bid
  - restrictions on offers, as a function of:
    - bidder's own previous bid
    - auction state (others' bids)
    - eligibility (e.g., budget constraints)
    - expiration, withdrawal, replacement
- rules for what information is revealed
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
  - when to reveal what information to whom
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for clearing
  - when to clear
    - at intervals
    - on each bid
    - after a period of inactivity
  - allocation (who gets what)
  - payment (who pays what)

Lecture Overview

Intuitive comparison of 5 auctions

	English	Dutch	Japanese	1st-Price	2nd-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2nd-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

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- How should agents bid in these auctions?

Fun Game

- Valuation models:
  - the most important one: IPV
  - valuations are iid draws from some commonly-known distribution
    - do you see how we can write this as a Bayesian game?

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Price Discovery	yes	no	yes	no	no
Regret	no	yes	no	yes	no

Lecture Overview

Second-Price

Theorem

Truth-telling is a dominant strategy in a second-price auction.

- In fact, we know this already (do you see why?)
- However, we'll look at a simpler, direct proof.

Second-Price proof

Theorem

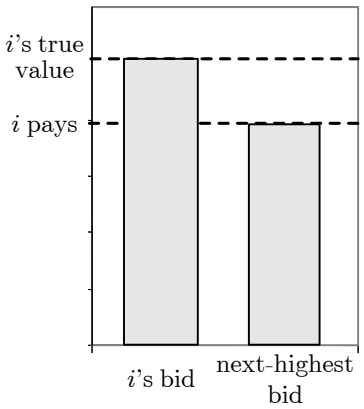
Truth-telling is a dominant strategy in a second-price auction.

Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

1. Bidding honestly, i would win the auction
2. Bidding honestly, i would lose the auction

Second-Price proof (2)



- Bidding honestly, i is the winner

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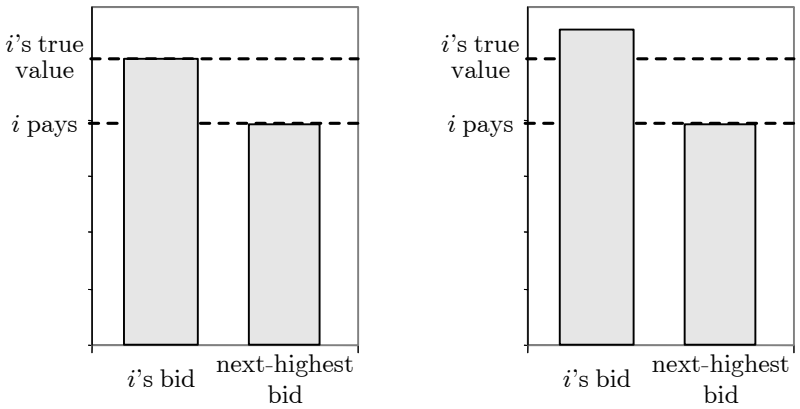

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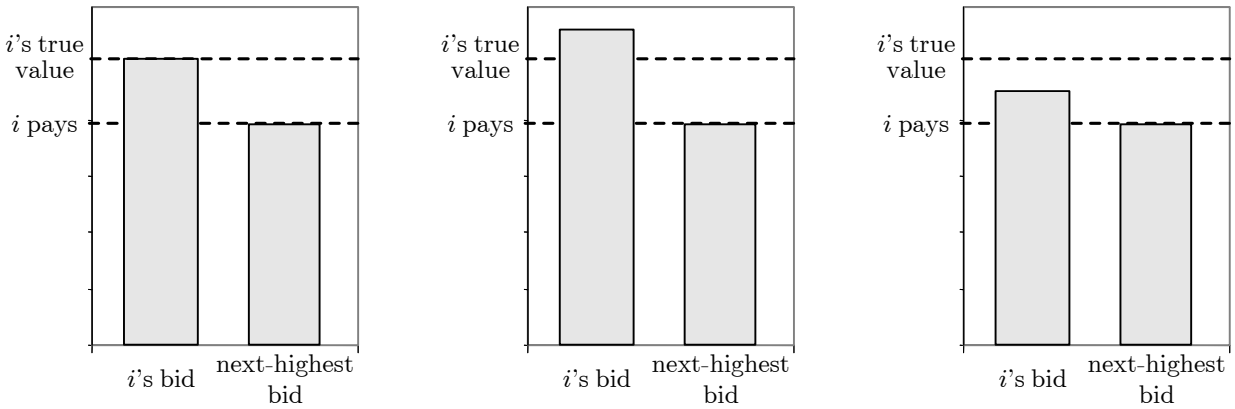

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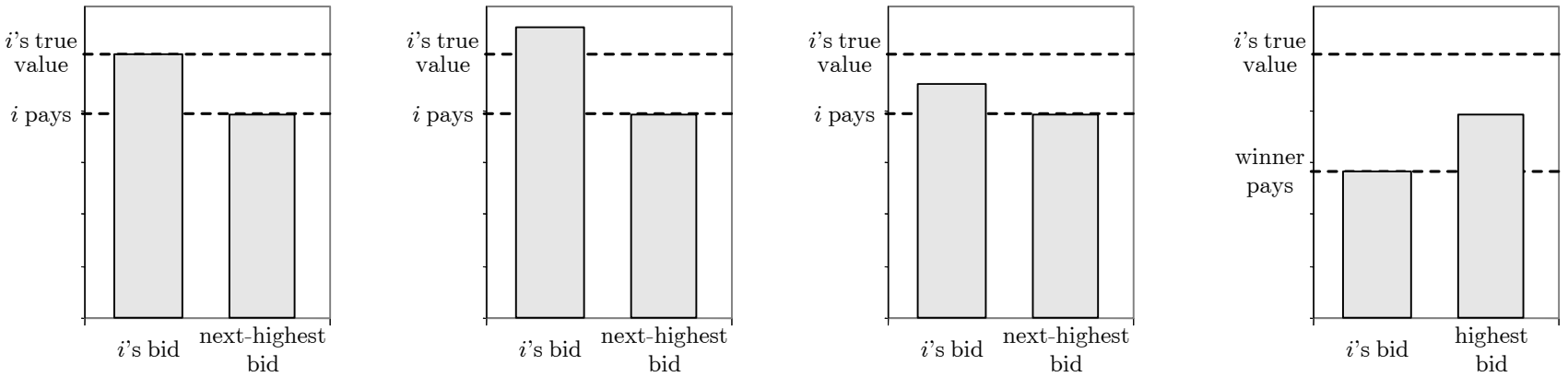

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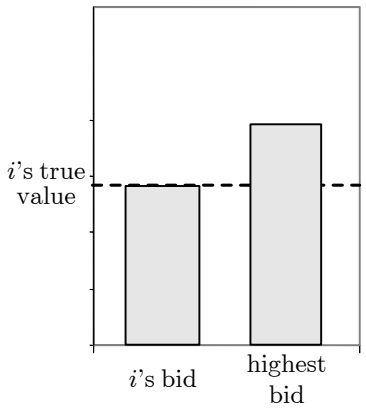
Second-Price proof (3)



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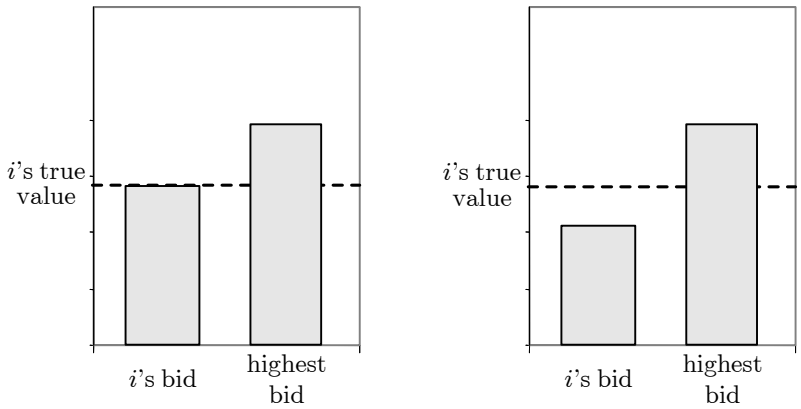

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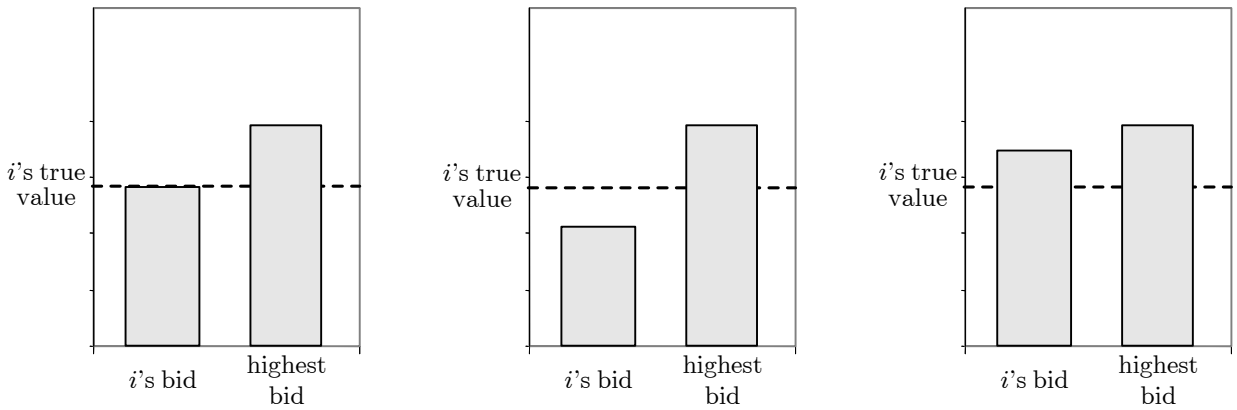

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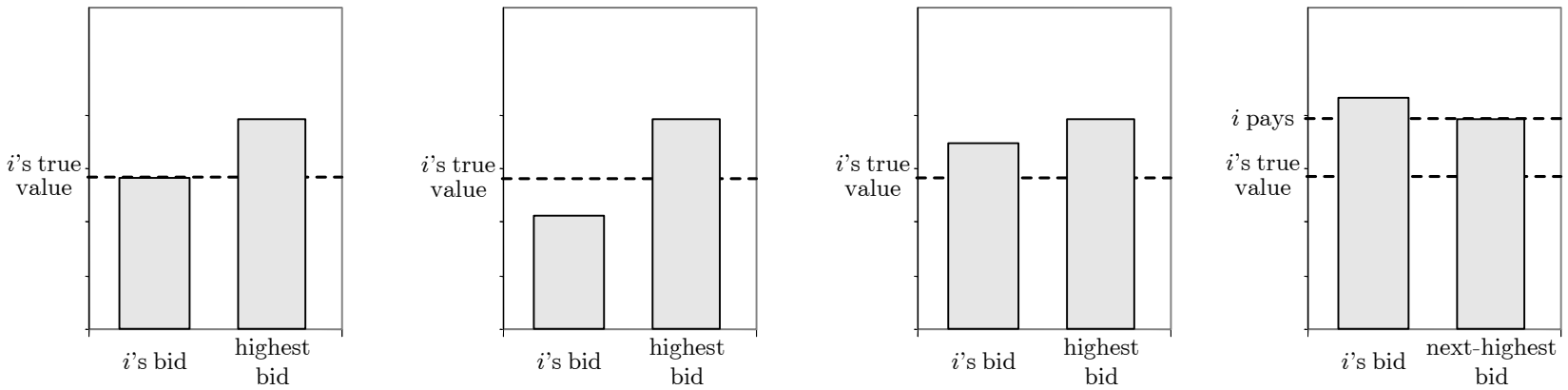

- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing
- If i bids higher, he will either still lose and still pay nothing. . . or win and pay more than his valuation.

English and Japanese auctions

- A much more complicated strategy space
  - extensive form game
  - bidders are able to condition their bids on information revealed by others
  - in the case of English auctions, the ability to place jump bids
- intuitively, though, the revealed information doesn't make any difference in the IPV setting.

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Theorem

Under the independent private values model (IPV), it is a dominant strategy for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.