CPSC 532L, Winter 2011
Final Exam

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1. **[25 points] Game Theory: 1D Battleship**

   Consider a simplified one-dimensional game of Battleship. Initially, player one places a ship of length ℓ_s into a space of water with length ℓ_w. Thus, his strategy is to choose an integer x_1 where $1 \leq x_1 \leq \ell_w - \ell_s + 1$. (This action is not observed by player two.) Player two then chooses an integer point x_2 where $1 \leq x_2 \leq \ell_w$. If this point touches the ship (i.e., if $x_1 \leq x_2 \leq x_1 + \ell_s - 1$), then the game ends. Otherwise, player two chooses another point, continuing on until the ship is found. Player one's payoff is equal to the number of guesses that player two had to make. Player two's payoff is the negative of player one's (i.e., the game is zero sum).

   (a) **[5 points]** Draw the corresponding perfect-recall imperfect information extensive form game, for the case where $\ell_w = 3$ and $\ell_s = 1$. (You can leave out choice nodes in which the agent only has one choice.)

   (b) **[6 points]** Find a Nash equilibrium of this game for the case where $\ell_w = 2\ell_s$. What is the value of this game?

   (c) **[7 points]** What is the space complexity of representing 1D Battleship with $\ell_s = 1$ in the imperfect information extensive form representation? Write your answer in big-O notation, as a function of ℓ_w.

   (d) **[7 points]** Consider a general n-player imperfect information extensive form game with c choice nodes, each of which has exactly two children. What is the space complexity of representing such a game in the induced normal form, in terms of n and c?

2. **[25 points] Combinatorial Auctions: Display Advertising**

   Consider the following combinatorial auction scenario: a professor wants to supplement his income by auctioning off advertising space on his lecture slides. He decides to use VCG as his auction mechanism and to offer two goods: a top banner space and a side-bar space. Every advertiser's type has two parts: v_i, which specifies his utility for an allocation that satisfies him; and $f_i \in \{t, s, b\}$, which specifies what must happen for him to be satisfied. An agent with $f_i = t$ will be satisfied if he wins the top banner space, regardless of what happens to the side-bar space. An agent with $f_i = s$ will be satisfied if he wins the side-bar space, regardless of what happens to the top banner space. An agent with $f_i = b$ will only be satisfied if he wins both spaces. (He might only use one space, but he insists on not having any other ad shown alongside his.)

   (a) **[5 points]** Let v_{1s} denote the first-highest bid for slot s. (Similarly define v_{1t}, v_{1b}, v_{2s}, v_{2t} and v_{2b}). Express the VCG revenue as a function of these values. Assume that the values are zero if there is no corresponding bidder. Assume that the auctioneer breaks ties in favor of bidders who want both slots.

   (b) **[5 points]** Demonstrate (with a specific example) that VCG's revenue can decrease as these quantities increase.

   (c) **[5 points]** Consider a setting where one of the agents is capable of creating a second identity (at some very small cost α), and submitting bids using both identities. (If he does so he gets every good won by either identity, and must pay for both.) Demonstrate that VCG is no longer dominant strategy truthful in this case: i.e., that an agent can strictly improve his utility by creating a second identity and misreporting his preferences for one or both identities.

   (d) **[10 points]** Consider the following auction mechanism: allocate both goods to whichever advertiser has submitted the highest bid, and charge him the amount of the second highest bid (ignoring the f_i element of their types). Prove that this mechanism has the following properties:

      i. the agents have a dominant strategy of reporting truthfully, using only their true identities, and

      ii. in equilibrium, the mechanism generates at least half as much revenue as VCG would (if the agents actually reported truthfully to VCG).

3. **[21 points] Social Choice: Ranking with a Distance Measure**

   Let $d(\succ, \succ')$ denote the distance between two rankings $\succ$ and $\succ'$, defined as the number of pairs o, o' for which $o \succ o'$ and $o' \succ' o$. For example, $d(A \succ B \succ C \succ D, C \succ B \succ A \succ D) = 3$ because the rankings disagree on (A, B), (A, C) and (B, C). Let $\succ_i$ denote the preferences of agent i. Define the cost of a ranking $c(\succ \mid \succ_1, \succ_2, ...)$ as the total distance between that ranking and the preferences of the agents:

   $$c(\succ \mid \succ_1, \succ_2, ...) = \sum_{i=1}^{n} d(\succ_i, \succ).$$

   Now, we say that a social welfare function M is cost-minimizing if it chooses a ranking $\succ$ that minimizes the cost. That is,

   $$M(\succ_1, \succ_2, ...) \in \arg\min_{\succ} c(\succ \mid \succ_1, \succ_2, ...).$$

   (a) **[5 points]** Consider the following preferences:

   $$\begin{aligned} &D \succ_1 A \succ_1 B \succ_1 C \\ &D \succ_2 A \succ_2 C \succ_2 B \\ &C \succ_3 B \succ_3 D \succ_3 A \\ &C \succ_4 D \succ_4 A \succ_4 B \\ &C \succ_5 D \succ_5 A \succ_5 B \end{aligned}$$

   What outcome will Borda select? What ranking will M select?

   (b) **[8 points]** Prove that if a Condorcet winner exists, it will be ranked first by M. Similarly, prove that if a Condorcet loser exists, it will be placed last by M. (For simplicity, you may assume that the number of agents is odd.)

   (c) **[8 points]** Consider using M as a mechanism, where each agent reports a ranking and the outcome ranked highest by the social welfare function is chosen. (When there are multiple cost-minimizing rankings, assume that M ties breaks ties lexicographically, e.g., alphabetically). Demonstrate (by providing a counterexample) that this mechanism is not truthful, even for the case of three voters and three outcomes.

4. **[29 points] Mechanism Design: The Seller and the Psychic**

   Consider the following problem: a mechanism designer wants to know how likely it is that a given unfair coin will come up heads when it is next tossed. There is an psychic (agent 1) who knows the true probability p of the coin coming up heads. The designer could just offer to pay the psychic a flat fee, but then the psychic's utility would be the same, regardless of the probability that the psychic reports, $\hat{p}$. In order to induce the psychic to reveal the true p, the mechanism designer commits to the following mechanism: if the coin comes up heads, then the psychic will be paid $c + \log_2 \hat{p}$, and if the coin comes up tails, then the psychic will be paid $c + \log_2(1 - \hat{p})$.

   (a) **[6 points]** Prove that the psychic's utility is maximized by revealing the true p.

   (b) **[10 points]** Now consider a scenario where p, again known by the psychic, represents the probability that a buyer (agent 2) has a high valuation (200 rather than 100). Because p represents the psychic's private information, we can understand it as the psychic's type. The seller knows that the joint type distribution is as follows:

p	v_2	Probability of this type profile
1/6	200	1/10
1/6	100	5/10
3/4	200	3/10
3/4	100	1/10

   Design a deterministic mechanism with the following properties:

   i. truthful in Bayes-Nash equilibrium;
   ii. the psychic's payment is consistent with the $\log_2 p$ rule above;
   iii. in equilibrium the psychic and the buyer are guaranteed *ex post* utilities of at least one and zero respectively; and
   iv. maximizes revenue, subject to satisfying (i-iii).

   Specify your mechanism by filling out the following table. (Include at least two significant figures for the payments.) Explain why your mechanism satisfies each requirement.

p	v_2	Outcome	p_1	p_2
1/6	200			
1/6	100			
3/4	200			
3/4	100			

   (c) **[5 points]** What mechanism would the seller choose to maximize revenue if the psychic wasn't available? Does does having access to the psychic increase the seller's expected (total) revenue? If so, by how much?

   (d) **[8 points]** Given this setting, is it possible to create a mechanism that is truthful in dominant strategies (where the dominance can be weak, but not very weak)? Explain why or why not.

Academic Honesty Form

The only resources to which I referred were my own notes, the textbook *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, lecture slides from CPSC 532L, and assignments or solutions to assignments from CPSC 532L. I did not refer to any other resources, including web sites, books, or people. I worked alone, and did not consult with other students from the class.

Signed: ____

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