Breadth-first Search; Search with Costs

CPSC 322 – Search 3

Textbook §3.5

Lecture Overview

Graph Search Algorithm

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s\rangle : s$ is a start node$\}$;
while $frontier$ is not empty:
select and **remove** path $\langle n_0, \ldots, n_k\rangle$ from $frontier$;
if $goal(n_k)$
return $\langle n_0, \ldots, n_k\rangle$;
for every neighbor n of n_k
add $\langle n_0, \ldots, n_k, n\rangle$ to $frontier$;
end while

- After the algorithm returns, it can be asked for more answers and the procedure continues.
- Which value is selected from the frontier defines the search strategy.
- The $neighbor$ relationship defines the graph.
- The $goal$ function defines what is a solution.

Depth-first Search

- Depth-first search treats the frontier as a stack
  - It always selects one of the last elements added to the frontier.

- Complete when the graph has no cycles and is finite
- Time complexity is $O(b^m)$
- Space complexity is $O(bm)$

DFS Example

- `http://aispace.org/search/`
- "simple tree graph"

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- When is DFS inappropriate?
  - some paths have infinite length
  - the graph contains cycles
  - some solutions are very deep, while others are very shallow

Lecture Overview

Breadth-first Search

- Breadth-first search treats the frontier as a queue
  - it always selects one of the earliest elements added to the frontier.

- Example:
  - the frontier is $[p_1, p_2, \ldots, p_r]$
  - neighbours of p_1 are $\{n_1, \ldots, n_k\}$
- What happens?
  - p_1 is selected, and tested for being a goal.
  - Neighbours of p_1 follow p_r at the end of the frontier.
  - Thus, the frontier is now $[p_2, \ldots, p_r, (p_1, n_1), \ldots, (p_1, n_k)]$.
  - p_2 is selected next.

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  - The order in which we examine nodes (BFS or DFS) makes no difference to the worst case: search is unconstrained by the goal.

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- What is the space complexity?
  - Space complexity is $O(b^m)$: we must store the whole frontier in memory

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  - although all solutions may not be shallow, at least some are
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- When is BFS inappropriate?
  - space is limited
  - all solutions tend to be located deep in the tree
  - the branching factor is very large