Logic: Datalog

CPSC 322 – Logic 6

Textbook §12.2

Lecture Overview

Top-down Ground Proof Procedure

Idea: search backward from a query to determine if it is a logical consequence of KB.

An answer clause is of the form:

$$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$$

The SLD Resolution of this answer clause on atom a_i with the clause:

$$a_i \leftarrow b_1 \wedge \ldots \wedge b_p$$

is the answer clause

$$yes \leftarrow a_1 \wedge \cdots \wedge a_{i-1} \wedge b_1 \wedge \cdots \wedge b_p \wedge a_{i+1} \wedge \cdots \wedge a_m.$$

Derivations

- An answer is an answer clause with $m = 0$. That is, it is the answer clause $yes \leftarrow$.
- A derivation of query "$?q_1 \wedge \ldots \wedge q_k$" from KB is a sequence of answer clauses $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that
  - γ_0 is the answer clause $yes \leftarrow q_1 \wedge \ldots \wedge q_k$,
  - γ_i is obtained by resolving γ_{i-1} with a clause in KB, and
  - γ_n is an answer.

Top-down definite clause interpreter

To solve the query $?q_1 \land \ldots \land q_k$:

$ac :=$ "$yes \leftarrow q_1 \land \ldots \land q_k$"
repeat
  select atom a_i from the body of ac;
  choose clause C from KB with a_i as head;
  replace a_i in the body of ac by the body of C
until ac is an answer.

Recall:

- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives. select
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

Lecture Overview

Objects and Relations

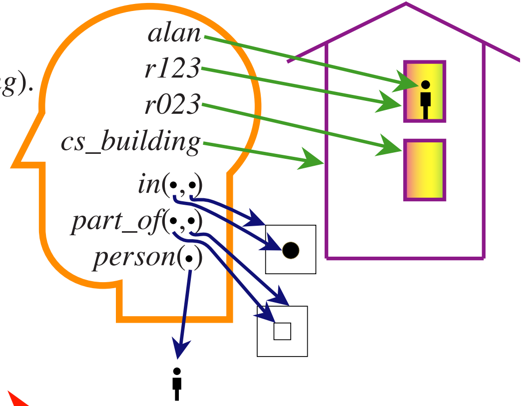
- It is useful to view the world as consisting of objects and relationships between these objects.
- Often the propositions we spoke about before can be condensed into a much smaller number of propositions if they are allowed to express relationships between objects and/or functions of objects.
- Thus, reasoning in terms of objects and relationships can be simpler than reasoning in terms of features, as you can express more general knowledge using logical variables.

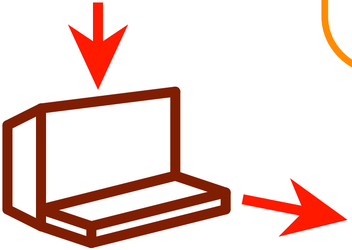
Using an RRS

1. Begin with a task domain.
2. Distinguish those objects you want to talk about.
3. Determine what relationships you want to represent.
4. Choose symbols in the computer to denote objects and relations.
5. Tell the system knowledge about the domain.
6. Ask the system questions.

Example Domain for an RRS

in(alan,r123).
part_of(r123,cs_building).
in(X,Y) ←
    part_of(Z,Y) ∧
    in(X,Z).





in(alan,cs_building)

Representational Assumptions of Datalog

- An agent's knowledge can be usefully described in terms of *individuals* and *relations* among individuals.
- An agent's knowledge base consists of *definite* and *positive* statements.
- The environment is *static*.
- There are only a finite number of individuals of interest in the domain. Each individual can be given a unique name.

$\Longrightarrow$ Datalog

Lecture Overview

Syntax of Datalog

Definition (variable)

A variable starts with upper-case letter.

Definition (constant)

A constant starts with lower-case letter or is a sequence of digits.

Definition (term)

A term is either a variable or a constant.

Definition (predicate symbol)

A predicate symbol starts with lower-case letter.

Syntax of Datalog (cont)

Definition (atom)

An atomic symbol (atom) is of the form p or $p(t_1, \ldots, t_n)$ where p is a predicate symbol and t_i are terms.

Definition (definite clause)

A definite clause is either an atomic symbol (a fact) or of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_m}_{\text{body}}$$

where a and b_i are atomic symbols.

Definition (knowledge base)

A knowledge base is a set of definite clauses.

Example Knowledge Base

$$
\begin{aligned}
&in(alan, R) \leftarrow \\
&\qquad teaches(alan, cs322) \land \\
&\qquad in(cs322, R). \\
&grandfather(william, X) \leftarrow \\
&\qquad father(william, Y) \land \\
&\qquad parent(Y, X). \\
&slithy(toves) \leftarrow \\
&\qquad mimsy \land borogroves \land \\
&\qquad outgrabe(mome, Raths).
\end{aligned}
$$

Lecture Overview

Semantics: General Idea

- Recall: a semantics specifies the meaning of sentences in the language.
  - ultimately, we want to be able to talk about which sentences are true and which are false
- In propositional logic, all we needed to do in order to come up with an interpretation was to assign truth values to atoms
- For Datalog, an interpretation specifies:
  - what objects (individuals) are in the world
  - the correspondence between symbols in the computer and objects & relations in world
    - which constants denote which individuals
    - which predicate symbols denote which relations (and thus, along with the above, which sentences will be true and which will be false)

Formal Semantics

Definition (interpretation)

An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- D, the domain, is a nonempty set. Elements of D are individuals.
- ϕ is a mapping that assigns to each constant an element of D. Constant c denotes individual $\phi(c)$.
- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{$*TRUE*, *FALSE*$\}$.

Example Interpretation

Constants: *phone*, *pencil*, *telephone*.
Predicate Symbol: *noisy* (unary), *left_of* (binary).

- $D = \{$✂, ☎, ✎$\}$.
  - These are actually objects in the world, not symbols
- $\phi(phone) =$ ☎, $\phi(pencil) =$ ✎, $\phi(telephone) =$ ☎.
- $\pi(noisy)$:

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

$\pi(left\_of)$:

⟨✂, ✂⟩	*FALSE*	⟨✂, ☎⟩	*TRUE*	⟨✂, ✎⟩	*TRUE*
⟨☎, ✂⟩	*FALSE*	⟨☎, ☎⟩	*FALSE*	⟨☎, ✎⟩	*TRUE*
⟨✎, ✂⟩	*FALSE*	⟨✎, ☎⟩	*FALSE*	⟨✎, ✎⟩	*FALSE*

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.
- The constants do not have to match up one-to-one with members of the domain. Multiple constants can refer to the same object, and some objects can have no constants that refer to them.
- $\pi(p)$ specifies whether the relation denoted by the n-ary predicate symbol p is true or false for each n-tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.
    - this was the situation in propositional logic

Truth in an interpretation

Definition (truth in an interpretation)

- A constant c denotes in I the individual $\phi(c)$.
- Ground (variable-free) atom $p(t_1, \ldots, t_n)$ is
  - *true* in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) =$ *TRUE*, where t_i denotes t'_i in interpretation I and
  - *false* in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) =$ *FALSE*.
- Ground clause $h \leftarrow b_1 \wedge \ldots \wedge b_m$ is
  - *false* in interpretation I if h is *false* in I and each b_i is *true* in I, and item *true* in interpretation I otherwise.
  - A knowledge base, KB, is *true* in interpretation I if and only if every clause in KB is *true* in I.

- Notice that truth values are only associated with predicates (atomic symbols; clauses), not variables and constants!

Example Truths

In the interpretation given before:

$$noisy(phone)$$

Example Truths

In the interpretation given before:

$noisy(phone)$ *true*

$noisy(telephone)$

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	*false*
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	

Example Truths

In the interpretation given before:

$noisy(phone)$	*true*
$noisy(telephone)$	*true*
$noisy(pencil)$	*false*
$left\_of(phone, pencil)$	*true*
$left\_of(phone, telephone)$	*false*
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	*true*
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	*false*
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	*true*

Variables

How do we determine the truth value of a clause that includes variables?

Definition (variable assignment)

A variable assignment is a function from variables into the domain.

- Given an interpretation and a variable assignment, each term denotes an individual and each clause is either true or false.
- A clause containing variables is true in an interpretation if it is true for all variable assignments.
  - Variables are universally quantified in the scope of a clause.

Models and logical consequences

Definition (model)

A model of a set of clauses is an interpretation in which all the clauses are *true*.

Definition (logical consequence)

If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.

- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.