Uninformed Search

CPSC 322 Lecture 5

September 14, 2007
Textbook §2.4

Lecture Overview

1. Graph Search
2. Searching
3. Depth-First Search

Search

- What we want to be able to do:
  - find a solution when we are not given an algorithm to solve a problem, but only a specification of what a solution looks like
  - idea: search for a solution

Definition (search problem)

A search problem is defined by

- A set of states
- A start state
- A goal state or goal test
  - a boolean function which tells us whether a given state is a goal state
- A successor function
  - a mapping from a state to a set of new states

Abstract Definition

How to search

- Start at the start state
- Consider the different states that could be encountered by moving from a state that has been previously expanded
- Stop when a goal state is encountered

To make this more formal, we'll need to talk about graphs...

Search Graphs

Definition (graph)

A graph consists of

- a set N of nodes;
- a set A of ordered pairs of nodes, called arcs or edges.

- Node n_2 is a neighbor of n_1 if there is an arc from n_1 to n_2.
  - i.e., if $\langle n_1, n_2 \rangle \in A$

Definition (path)

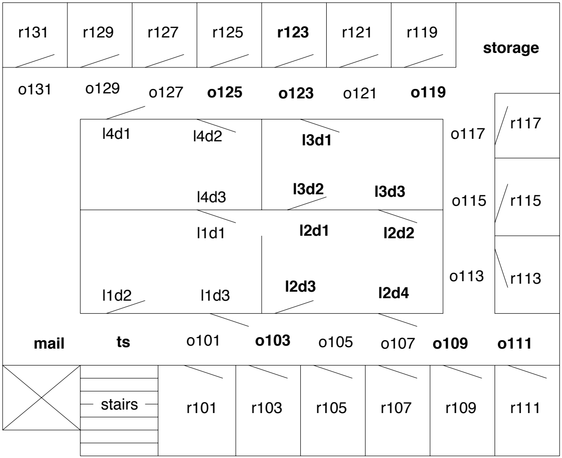
A path is a sequence of nodes $\langle n_0, n_1, \ldots, n_k \rangle$ such that $\langle n_{i-1}, n_i \rangle \in A$.

Definition (solution)

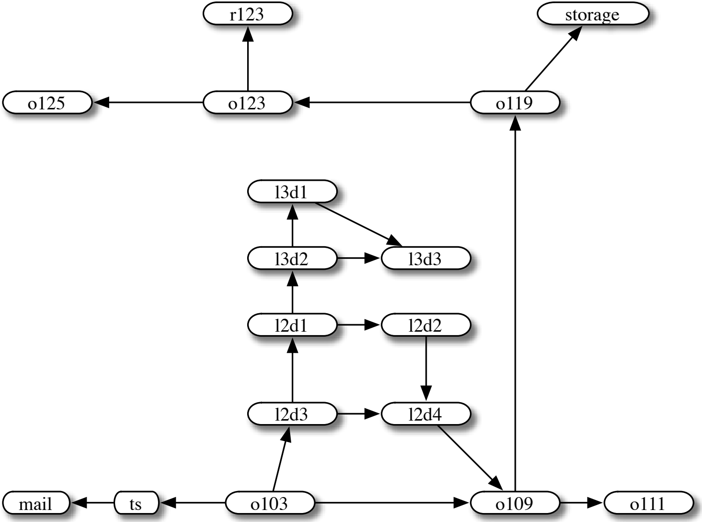
Given a start node and a set of goal nodes, a solution is a path from the start node to a goal node.

Example Domain for the Delivery Robot

The agent starts outside room 103,
and wants to end up inside room 123.



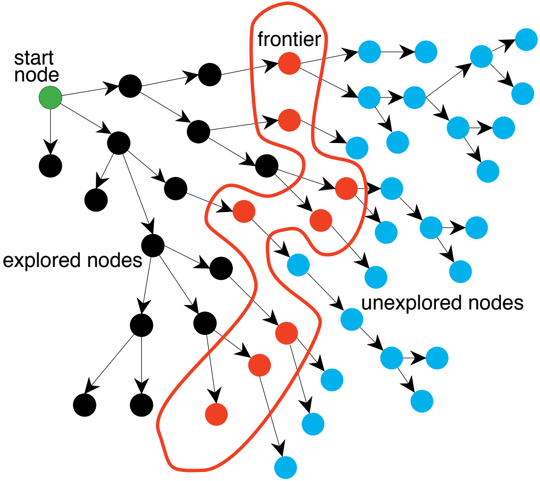
Example Graph for the Delivery Robot



Graph Searching

- Generic search algorithm: given a graph, start nodes, and goal nodes, incrementally explore paths from the start nodes.
- Maintain a frontier of paths from the start node that have been explored.
- As search proceeds, the frontier expands into the unexplored nodes until a goal node is encountered.

Problem Solving by Graph Searching



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- The way in which the frontier is expanded defines the search strategy.

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Graph Search Algorithm

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s \rangle : s$ is a start node$\}$;
while $frontier$ is not empty:
  select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from $frontier$;
  if $goal(n_k)$
    return $\langle n_0, \ldots, n_k \rangle$;
  for every neighbor n of n_k
    add $\langle n_0, \ldots, n_k, n \rangle$ to $frontier$;
end while

- After the algorithm returns, it can be asked for more answers and the procedure continues.
- Which value is selected from the frontier defines the search strategy.
- The $neighbor$ relationship defines the graph.
- The $goal$ function defines what is a solution.

Branching Factor

Definition (forward branching factor)

The forward branching factor of a node is the number of arcs going out of that node.

Definition (backward branching factor)

The backward branching factor of a node is the number of arcs going into the node.

- If the forward branching factor of every node is b and the graph is a tree, how many nodes are exactly n steps away from the start node?

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  - b^n nodes.
- We'll assume that all branching factors are finite.

Comparing Algorithms

Definition (complete)

A search algorithm is complete if, whenever at least one solution exists, the algorithm is guaranteed to find a solution within a finite amount of time

Definition (time complexity)

The time complexity of a search algorithm is an expression for the worst-case amount of time it will take to run, expressed in terms of the maximum path length m and the maximum branching factor b.

Definition (space complexity)

The space complexity of a search algorithm is an expression for the worst-case amount of memory that the algorithm will use, expressed in terms of m and b.

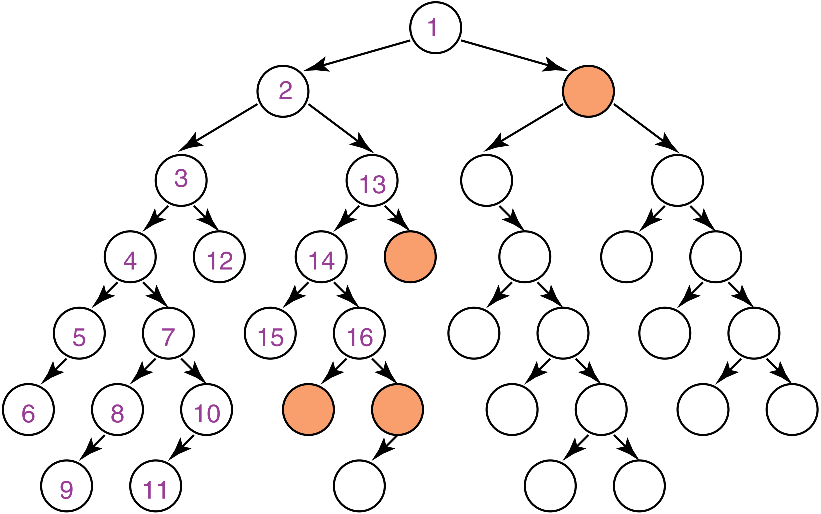
Lecture Overview

Depth-first Search

- Depth-first search treats the frontier as a stack
- It always selects one of the last elements added to the frontier.

- Example:
  - the frontier is $[p_1, p_2, \ldots, p_r]$
  - neighbours of p_1 are $\{n_1, \ldots, n_k\}$
- What happens?
  - p_1 is selected, and tested for being a goal.
  - Neighbours of p_1 replace p_1 at the beginning of the frontier.
  - Thus, the frontier is now $[(p_1, n_1), \ldots, (p_1, n_k), p_2, \ldots, p_r]$.
  - p_2 is only selected when all paths extending p_1 have been explored.

Illustrative Graph — Depth-first Search Frontier



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  - Search is unconstrained by the goal until it happens to stumble on the goal.
- What is the space complexity?
  - Space complexity is $O(bm)$: the longest possible path is m, and for every node in that path must maintain a fringe of size b.