AMS Verification using Coho-Reach

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What do we want to verify?

- **Global convergence**
- Small signal response
- Macro-models match schematic and layout
- Parasitic modeling is detailed enough
- Robust to PVT variation
- ...

How do we verify convergence?

- Weakly non-linear models.
- Coho-Reach to show convergence.
- Decompose verification into a few, simple lemmas.
Global Convergence

- Starting from **anywhere** does the AMS circuit reach its intended operating region.
- Establishes proper start-up.
- Establishes proper mode-changes.
- Large signal domain non-linearities must be considered.
- Once the circuit is in its intended operating region, traditional, small-signal methods, e.g., **PAC** and/or **QPAC** can be used to verify behavior and performance.
- Global convergence failures can occur because:
  - Unmodeled circuit behaviors outside of intended linear operating region:
    - Circuit failures
    - Unexpected non-linearities
  - Unexpected mode-change cycles.
Weakly non-linear models

- Linear models,
  \[ \dot{x} = Ax + b \]
  are easy to analyse – simple, closed form solutions.
- Non-linear models can be over-approximated by linear differential inclusions.
- When is a model weakly non-linear?
Linear models are easy to analyse.

Non-linear models can be over-approximated by linear differential inclusions:

\[ \dot{x} \in Ax + b \oplus \langle \text{NonLin} \rangle \]

- Linear differential inclusions are amenable to analysis similar to linear differential equations.
- Models can include mode-transitions.

When is a model weakly non-linear?
Weakly non-linear models

- Linear models are easy to analyse.
- Non-linear models can be \textit{over-approximated} by linear differential inclusions.
- When is a model \textbf{weakly} non-linear?
  - The \textit{designer’s} view: when linear behavior was “intended” [KJLH09].
  - The \textit{verifier’s} view: when a linear inclusion is precise enough to verify the desired property.
We omitted the Delta-Sigma modulator, low-pass filter, and linear regulator for simplicity.

- We believe all of these could be included using the same methods that we’ve used for the rest of the digital PLL.
Linear Inclusion for the digital PLL

\[ \dot{c} = g_1 \text{sign}(\theta), \] Capacitance, mode switching

\[ \dot{v} = g_2(c_{\text{center}} - c) \] Control voltage

\[ \dot{\theta} = f_{\text{DCO}}(c, v) - f_{\text{ref}} - \gamma \theta \]

\[ f_{\text{DCO}} = \frac{1 + \alpha v}{1 + \beta c} f_0 \]

Saturation conditions:

- \( \dot{c} = 0 \) if \( c = c_{\text{min}} \) and \( \theta < 0 \) or \( c = c_{\text{max}} \) and \( \theta > 0 \).
- \( \dot{v} = 0 \) if \( v = v_{\text{min}} \) and \( c > c_{\text{center}} \) or \( v = v_{\text{max}} \) and \( c < c_{\text{center}} \).
Verification with Coho-Reach

Initially, $\theta \in [-2\pi, +2\pi]$.

- Coho-Reach’s Matlab API [Yan14] supports lemma verification including

$$pre \rightarrow condition \quad \xrightarrow{dynamics} \quad post \rightarrow condition$$
We use Coho-Reach to show:

- **Init** \(\xrightarrow{\text{DPLL}}\) \(c \text{ – saturated} \cup \text{leaving} \cup \text{zigzag}\)
- **c – saturated** \(\xrightarrow{\text{DPLL}}\) **leaving**
- **leaving** \(\xrightarrow{\text{DPLL}}\) **away} \subset \text{zigzag**
- **zigzag** \(\xrightarrow{\text{DPLL}}\) **Operating region**
Conclusions

- Global convergence ensures the circuit will reach its intended operating region.
  - Many AMS circuits can be weakly non-linear, hybrid systems.
  - Amenable to reachability analysis.
  - Stiffness is a concern: circuits can have multiple control loops operating with much different time constants.
  - Convergence to a switching surface is another consequence of AMS design.
    - This favors verification by simple reachability lemmas rather than automatic computation of fix-points (e.g. [WPYG13]).

- Once the circuit is in its intended operating region,
  - Small-signal methods such as PAC and QPAC can verify performance.
References


