Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 19

March, 1, 2021

Slide Sources
Raymond J. Mooney University of Texas at Austin

D. Koller, Stanford CS - Probabilistic Graphical Models
D. Page, Whitehead Institute, MIT

Several Figures from
"Probabilistic Graphical Models: Principles and Techniques" *D. Koller, N. Friedman* 2009

Lecture Overview

- Recap: Naïve Markov – Logistic regression (simple CRF)
- CRFs: high-level definition
- CRFs Applied to sequence labeling
- NLP Examples: Name Entity Recognition, joint POS tagging and NP segmentation
- CFR + deep learning Example

Conditional Random Fields (CRFs)

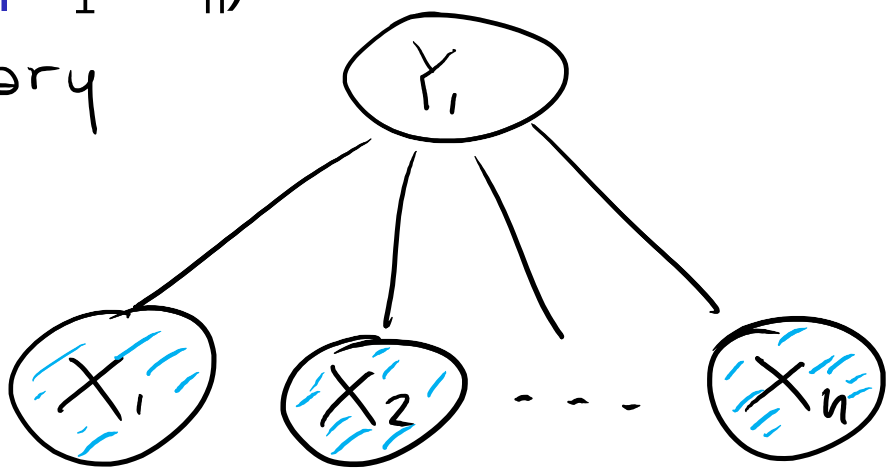
- Model $P(Y_1 .. Y_k \mid X_1 .. X_n)$
- Special case of Markov Networks where all the X_i are always observed

  - Simple case $P(Y_1 \mid X_1 \ldots X_n)$

all vars are binary

$Y_1 = \{0, 1\}$

$\forall i \; X_i = \{0, 1\}$



Some notation

exp and indicator function

$$\mathbb{1}(\text{Predicate}(x)) = \begin{cases} 1 & \text{if Pred}(x) \text{ is true} \\ 0 & \text{if Pred}(x) \text{ is false} \end{cases}$$

$$\exp(z) = e^{z}$$

Examples

$X = \{1, 2, 3, 4, 5\}$

set

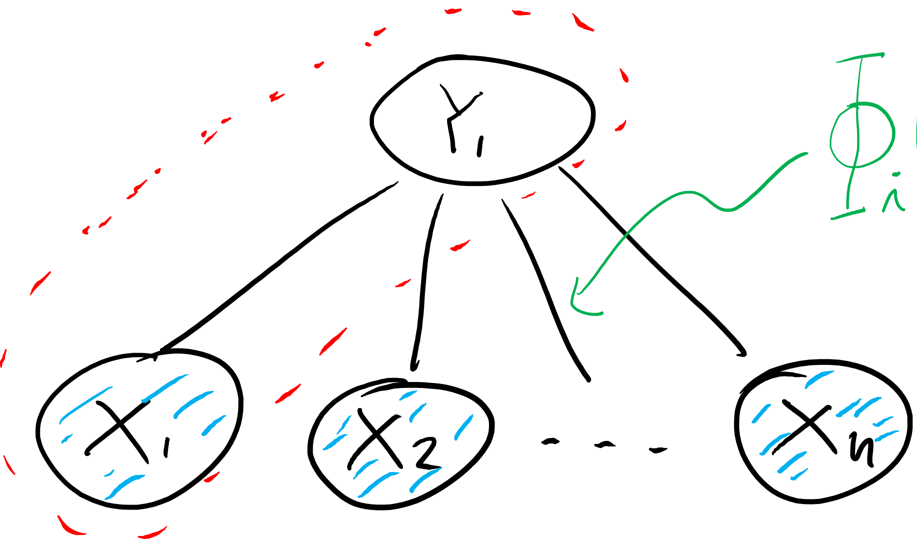
$$\sum_{x_i \in X} x_i = 15$$

$$\sum \mathbb{1}(\text{Even}(x_i))$$

$$\sum \mathbb{1}(\text{Prime}(x_i)) = 3$$

$= 2$

What are the Parameters?



$\Phi_i(X_i, Y_1) = \exp\{w_i * \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$

one such factor for each clique

also $\Phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y = 1\}\}$

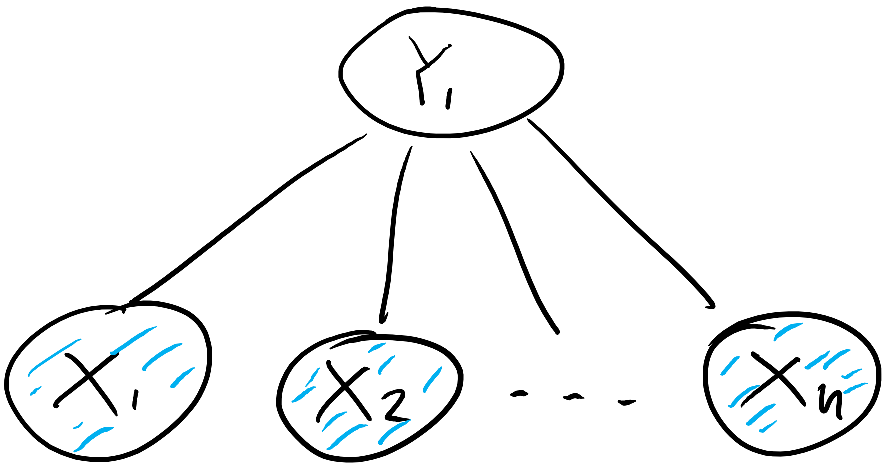
Example $w_2 = 1.5$ $\Phi_2(X_2, Y_1)$

X_2	Y_1	Φ_2
1	1	$e^{1.5}$
0	1	1
1	0	1
0	0	1

Example $w_0 = .4$

Y_1	Φ_0
0	1
1	$e^{.4}$

Let's derive the probabilities we need



To compute

$$P(\mathbf{Y_1} | X_1 \ldots X_n) = P(\mathbf{Y_1}, X_1 \ldots X_n) / P(X_1 \ldots X_n)$$

We compute

a

$$P(\mathbf{Y_1} = 1 | X_1 \ldots X_n) = P(\mathbf{Y_1} = 1, X_1 \ldots X_n) / \underline{P(X_1 \ldots X_n)}$$

$$P(Y_1 = 1, X_1 \ldots X_n) + P(Y_1 = 0, X_1 \ldots X_n)$$

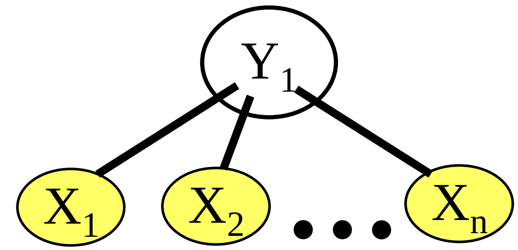
a b

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i * \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

how strongly $Y_1 = 1$ given that $X_i = 1$

$$\phi_0(Y_1) = \exp\{w_0 * \mathbb{1}\{Y_1 = 1\}\}$$



$$\tilde{P}(Y_1 = 1, x_1, x_2, \ldots, x_n) = \phi_0(Y_1) * \prod_{i=1}^{n} \phi_i(x_i, Y_1)$$

A. $e^{\sum_{1}^{n} w_i}$

B. $e^{w_0 + \sum_{1}^{n} w_i * x_i}$

D $e^{w_0 + \sum_{1}^{n} w_i}$

C. $e^{w_0 + \sum_{1}^{n} x_i}$

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

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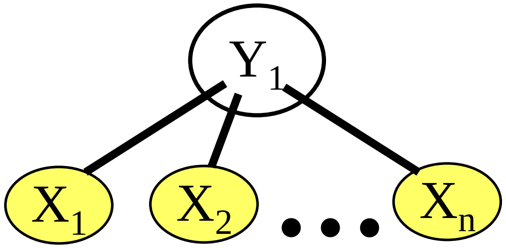


$$\tilde{P}(Y_1 = 1, x_1, x_2, \ldots, x_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(x_i, Y_1)$$

example

$$P(Y_1 = 1, X_1 = 0, X_2 = 1, X_3 = 1)$$

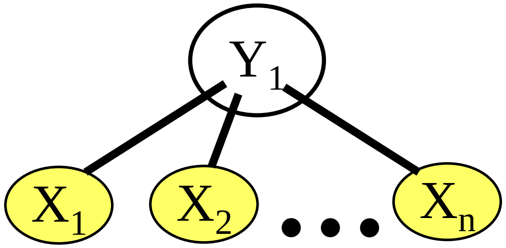
$$e^{w_0 * 1} * e^{w_1 * 0} * e^{w_2 * 1} * e^{w_3 * 1}$$

$$e^{w_0} * e^{w_1 * x_1} * e^{w_2 * x_2} * e^{w_3 * x_3} =$$

$$= \boxed{e^{w_0 + \sum w_i x_i}}$$

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$



$$\phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y_1 = 1\}\}$$

$$\tilde{P}(Y_1 = 0, x_1, x_2, \ldots, x_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(x_i, Y_1)$$

A. 1

B. e^{w_0}

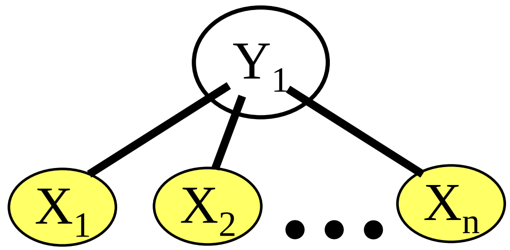
C. 0

D. $e^{\sum_{1}^{n} w_i}$

Let's derive the probabilities we need

a $\tilde{P}(Y_1 = 1, x_1, \ldots, x_n) = \exp(w_0 + \sum_{i=1}^{n} w_i x_i)$

b $\tilde{P}(Y_1 = 0, x_1, \ldots, x_n) = 1$



$$P(Y_1 = 1 \mid x_1, \ldots, x_n) = \frac{\tilde{P}(Y_1 = 1, x_1, \ldots, x_n)}{P(x_1, \ldots, x_n)}$$

sum of a and b

$$= \frac{\exp(w_0 + \sum w_i x_i)}{1 + \exp(w_0 + \sum w_i x_i)}$$

z

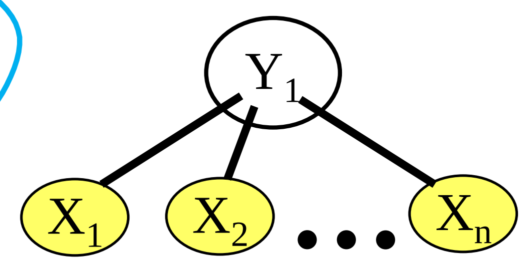
$$\frac{e^z}{1 + e^z} \quad \text{or} \quad \frac{1}{e^{-z} + 1}$$

sigmoid function

Let's derive the probabilities we need

a $\tilde{P}(Y_1 = 1, x_1, \ldots., x_n) = \exp(\underbrace{w_0 + \sum_{i=1}^{n} w_i x_i}_{z})$

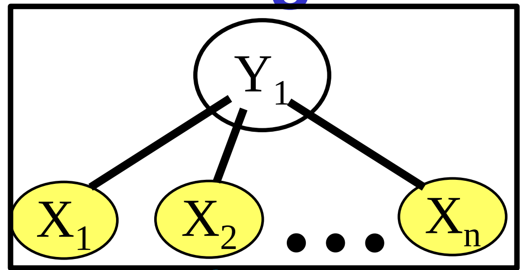
b $\tilde{P}(Y_1 = 0, x_1, \ldots., x_n) = 1$

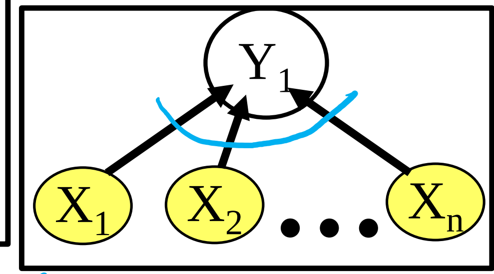


$$P(Y_1 = 1 \mid x_1, \ldots., x_n) = \frac{\tilde{P}(Y_1 = 1, x_1, \ldots, x_n)}{P(x_1, \ldots, x_n)} \leftarrow \text{sum of (a) and (b)}$$

$$= \boxed{\frac{e^{z}}{1 + e^{z}}} \cdot \frac{e^{-z}}{e^{-z}} \qquad \boxed{\frac{1}{e^{-z} + 1}}$$

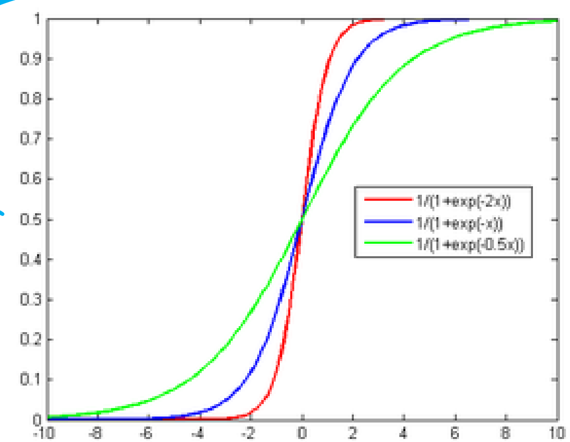
Sigmoid Function used in Logistic Regression

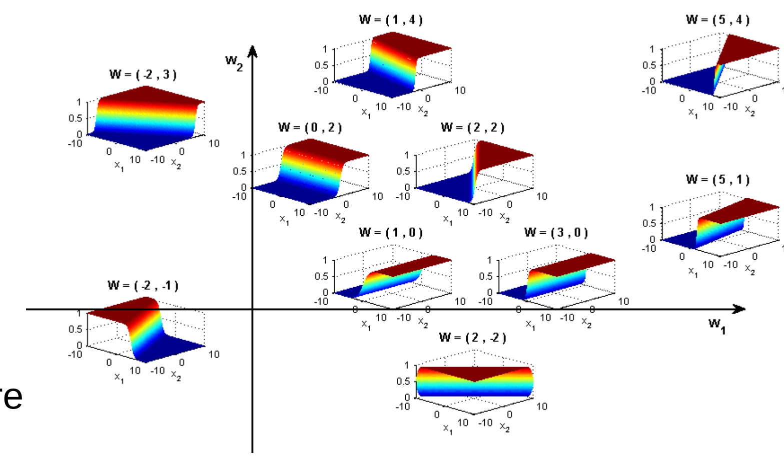




- Great practical interest
- Number of param w_i is linear instead of exponential in the number of parents
- Natural model for many real-world applications
- Naturally aggregates the influence of different parents

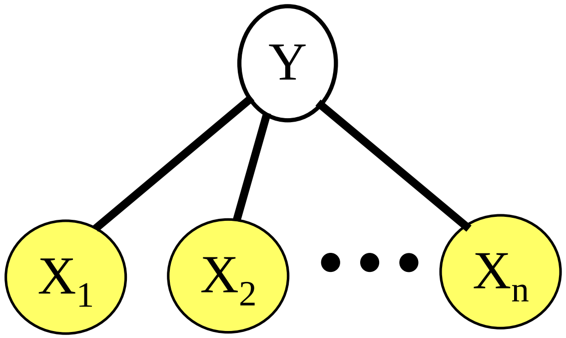
$\frac{1}{1+e^{-x}}$





Logistic Regression as a Markov Net (CRF)

Logistic regression is a simple Markov Net (a CRF) *aka* **naïve markov model**



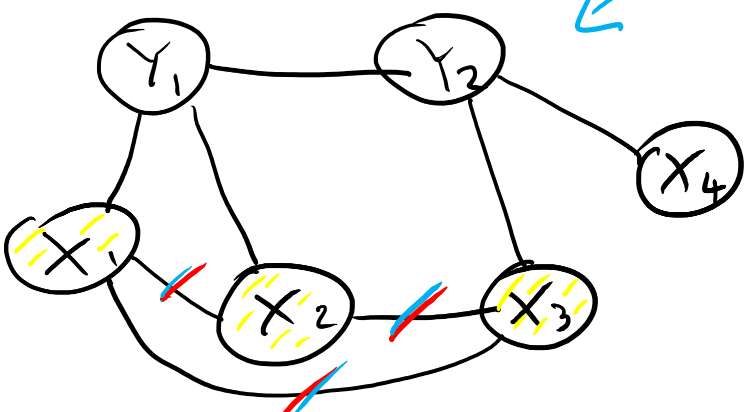
- But only models the **conditional distribution**, P(Y | $\boldsymbol{X}$) and not the full joint P(X,Y)

Let's generalize ….

Assume that you always observe a set of variables **X** = $\{X_1 \ldots X_n\}$ and you want to predict one or more variables **Y** = $\{Y_1 \ldots Y_k\}$

A **CRF** is an undirected graphical model whose nodes corresponds to **X** ∪ **Y**.

$\phi_1(D_1) \ldots \phi_m(D_m)$ represent the factors which annotate the network (but we disallow factors involving only vars in **X** – why?)



They would be

A. too large

B. constant

C. difficult to aquire

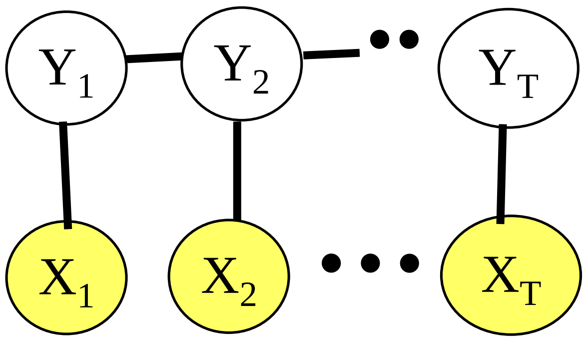
$$P(Y \mid X) = \frac{1}{Z(X)} \left(\prod_{i=1}^{m} \phi_i(D_i) \right)$$

$$Z(X) = \sum_{Y} \left(\prod_{i=1}^{m} \phi_i(D_i) \right)$$

Lecture Overview

- Recap: Naïve Markov – Logistic regression (simple CRF)
- CRFs: high-level definition
- **CRFs Applied to sequence labeling**
- NLP Examples: **Name Entity Recognition**, joint POS tagging and NP segmentation
- CFR + deep learning Example

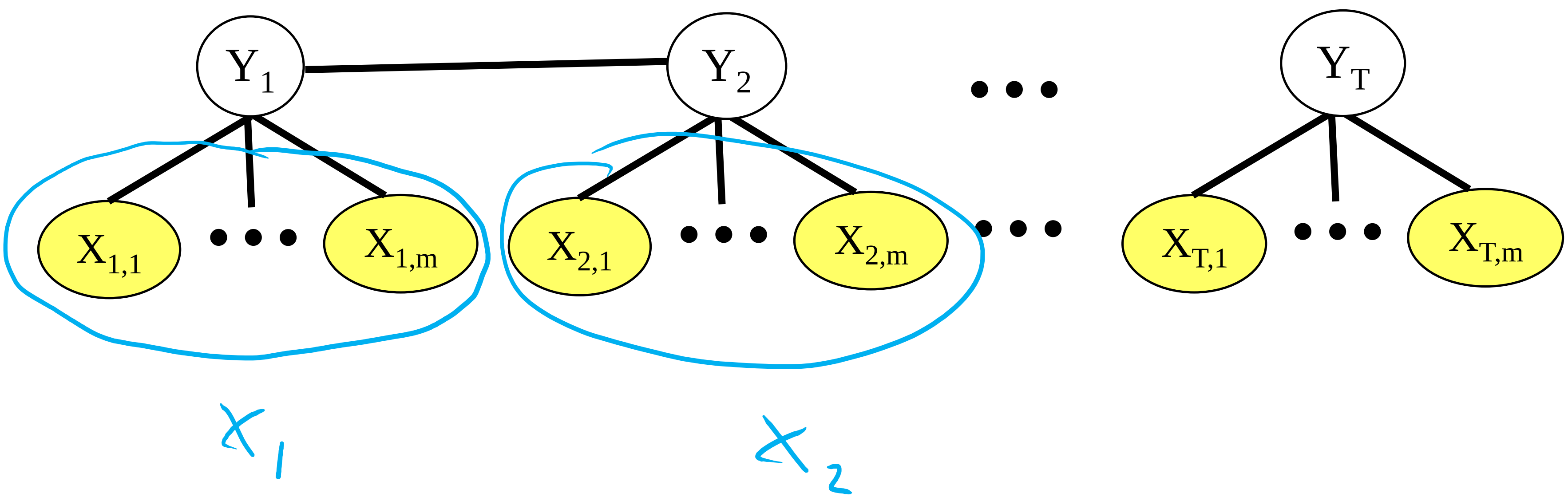
Sequence Labeling



Linear-chain CRF

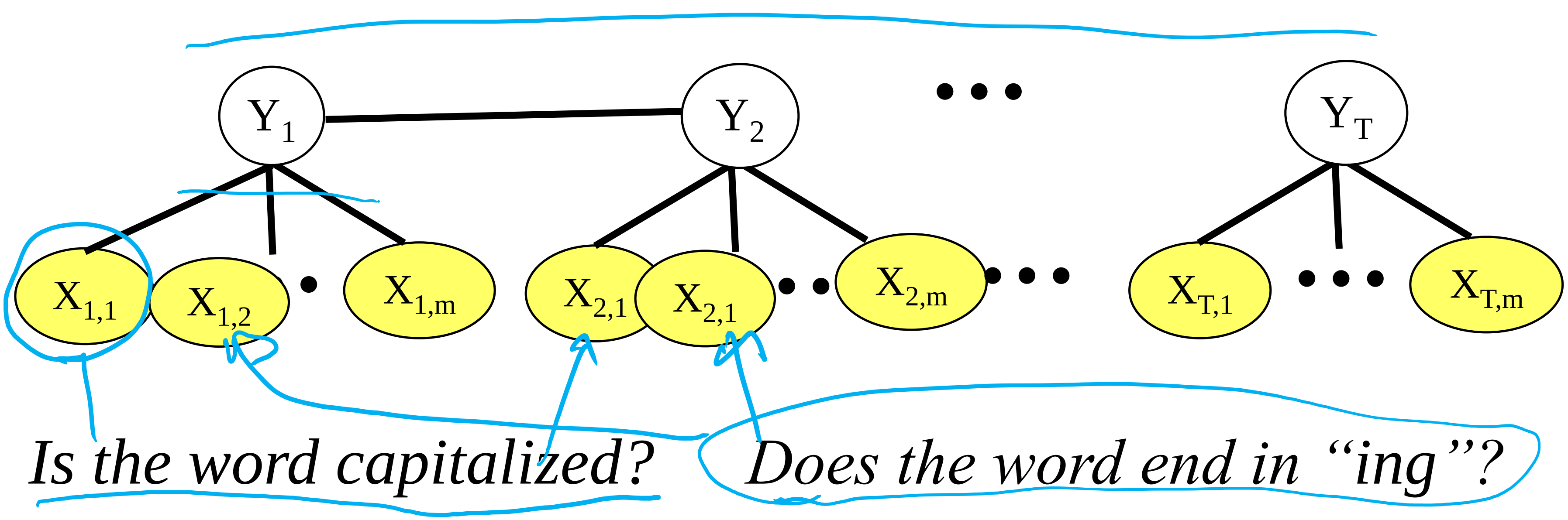
Increase representational Complexity: Adding Features to a CRF

- Instead of a single observed variable X_i we can model multiple features X_{ij} of that observation.



CRFs in Natural Language Processing

- One target variable Y for each word X, encoding the possible labels for X
- Each target variable is connected to a set of feature variables that capture properties relevant to the target distinction



Named Entity Recognition Task

- Entity often span multiple words *“British Columbia”*
- Type of an entity may not be apparent for individual words *“University of British Columbia”*
- Let’s assume three categories***: Person, Location, Organization***
- BIO notation (for sequence labeling)

possible labels B-PER I-PER B-LOC I-LOC
B-ORG I-ORG OTHER

O	B-ORG	I-ORG	I-ORG	I-ORG
The	University	of	British	Columbia

O	O	B-LOC	I-LOC
is	in	Vancouver	B.C.

Linear chain CRF parameters

With two factors "types" for each word

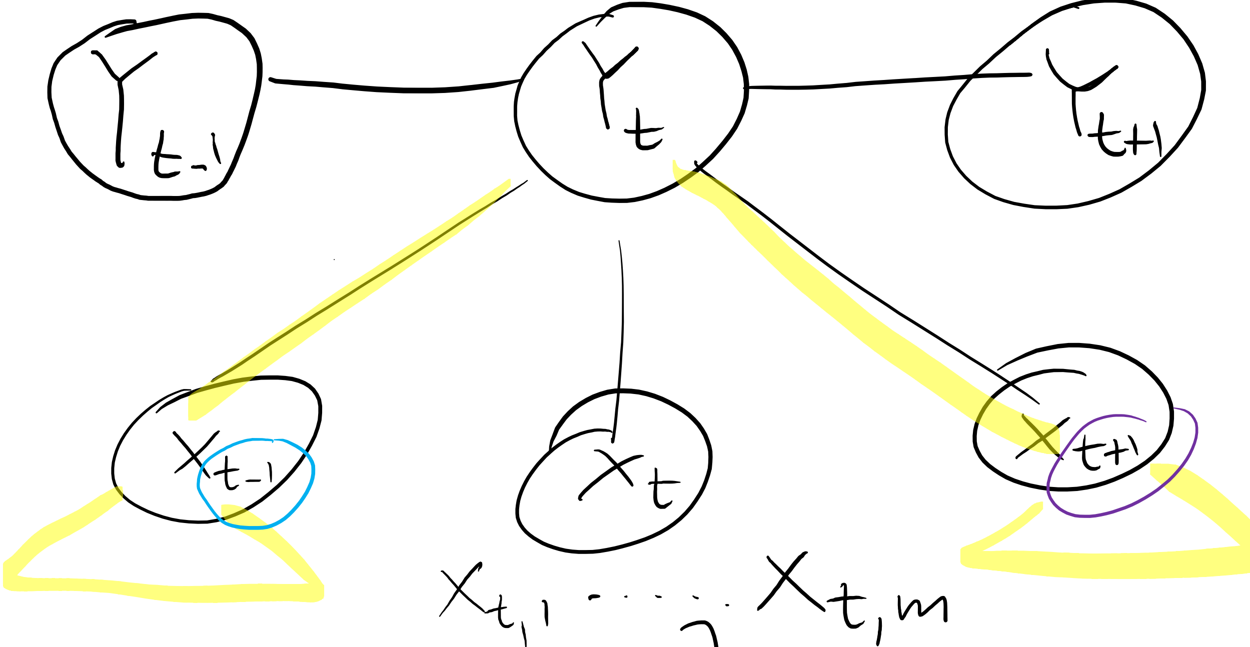
$\phi_t^1(Y_t, Y_{t-1})$ $\phi_t^1(Y_t, Y_{t+1})$ Dependency between neighboring target vars

$\phi_t^2(Y_t, X_1, .., X_T)$ Dependency between target variable and its context in the word sequence, which can include also **features of the words** (capitalized, appear in an atlas of location names, etc.)

Factors are similar to the ones for the Naïve Markov (logistic regression)

$$\phi_t(Y_t, X_{tk}) = \exp\{w_{tk} \times \mathbb{1}\{Y_t = \text{I-LOC}, X_{tk} = 1\}\}$$

appears in atlas of location names



$$\mathbb{1}\{Y_t = \text{I-ORG}, X_{t,k} = \text{"Times"}\}$$

$$\mathbb{1}\{Y_t = \text{I-PER}, X_{t+1,k} = \text{"spoke"}\}$$

$$\mathbb{1}\{Y_t = \text{I-PER}, X_{t-1,k} = \text{"Mrs."}\}$$

Features can also be

- The word
- Following word
- Previous word

More on features

Including features that are **conjunctions of simple features** increases accuracy

Y_t

$$\mathbb{1}\{Y_t = \text{I-PER}, X_{t+1,k} = \text{"spoke"}\}$$

$$\mathbb{1}\{Y_t = \text{I-PER}, X_{t-1,k} = \text{"Mrs."}\}$$

Total number of features can be 10^5-10^6

However features are sparse i.e. most features are 0 for most words

Linear-Chain Performance

Per-token/word accuracy in the high 90% range for many natural datasets

label is wrong for 2 words out of 9

Per-field precision and recall are more often around 80-95% , depending on the dataset. Entire Named Entity Phrase must be correct

only one is correct out of 2

O	B-ORG		I-ORG	B-LOC	I-LOC
The	University	of		British	Columbia

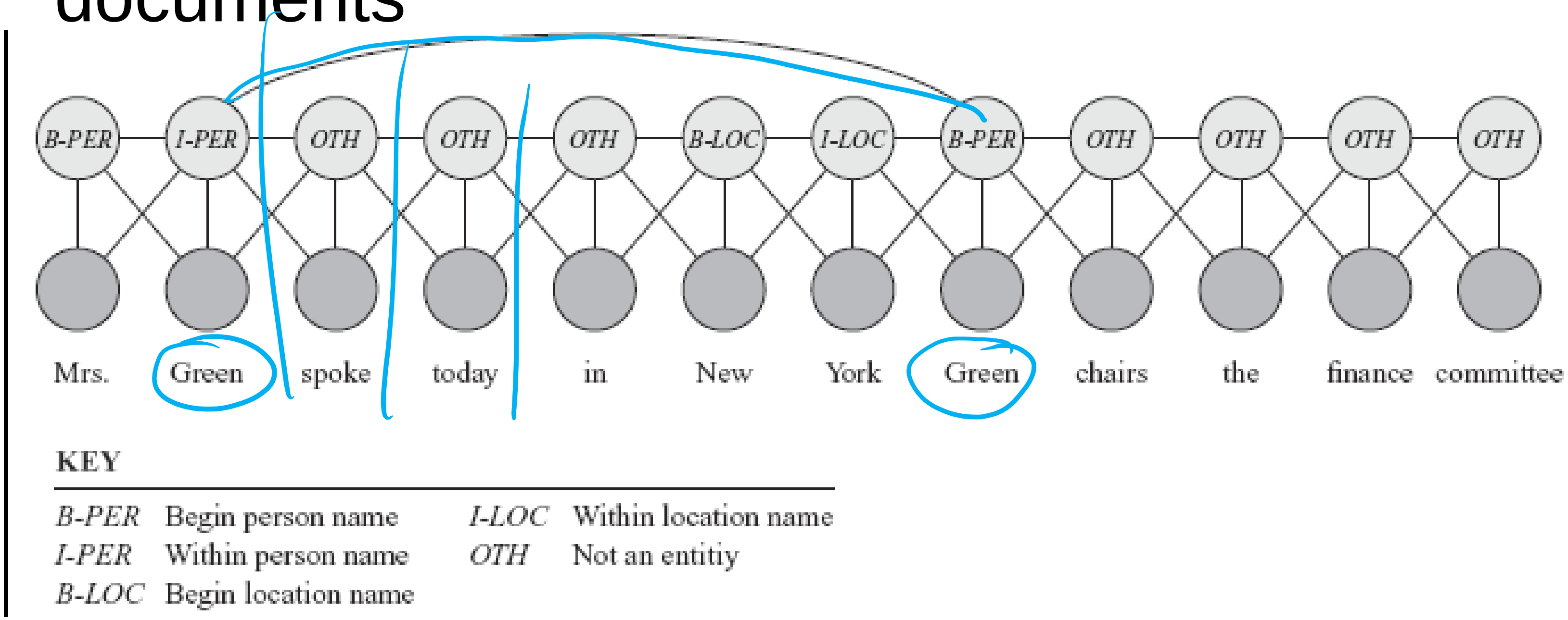
O	O	B-LOC	I-LOC
is	in	Vancouver	B.C.

	A.	B	C.
Per-word accuracy ?	1/2	7/9	7/9
Per-field precision ?	1/2	3/9	1/2

Skip-Chain CRFs

Include additional factors that connect non-adjacent target variables

E.g., When a word occur multiple times in the same documents



Graphical structure over Y can depend on the values of the Xs !

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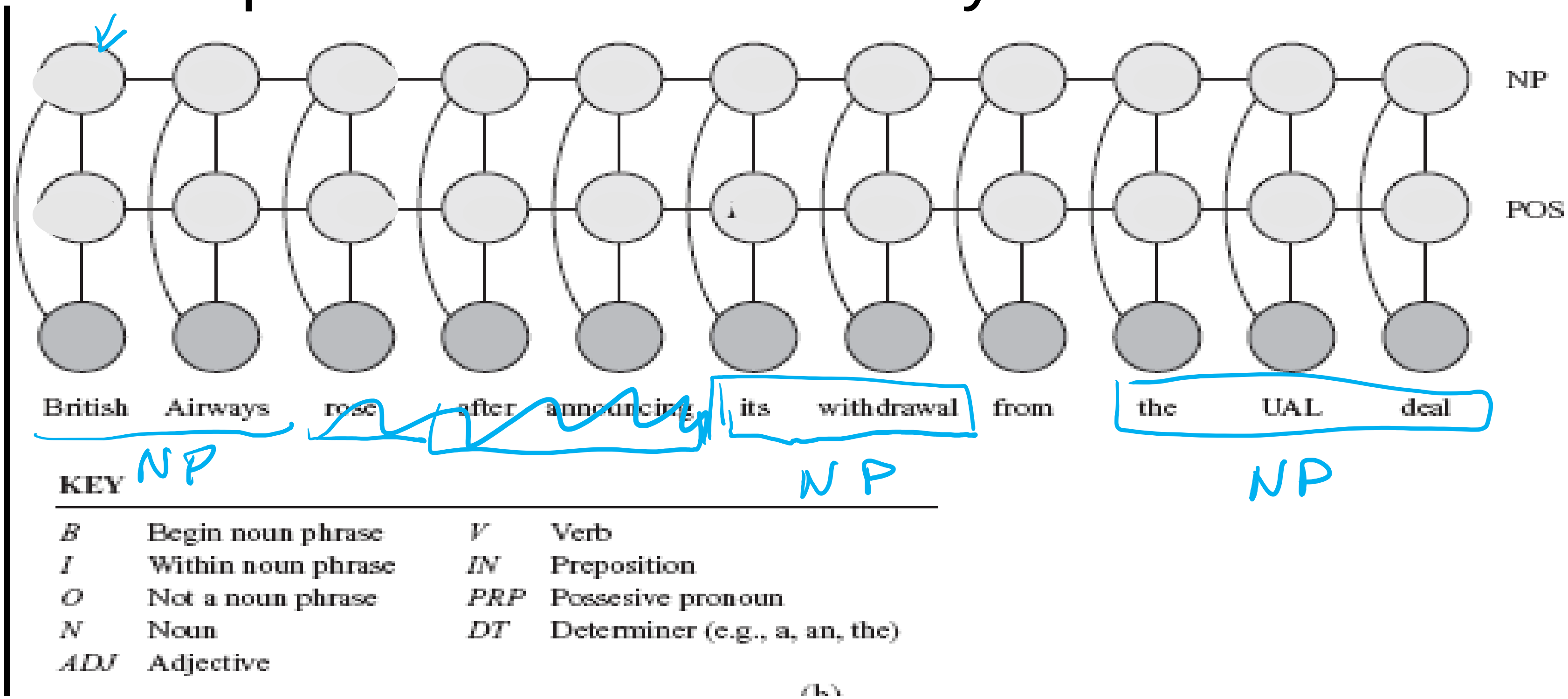
Coupled linear-chain CRFs

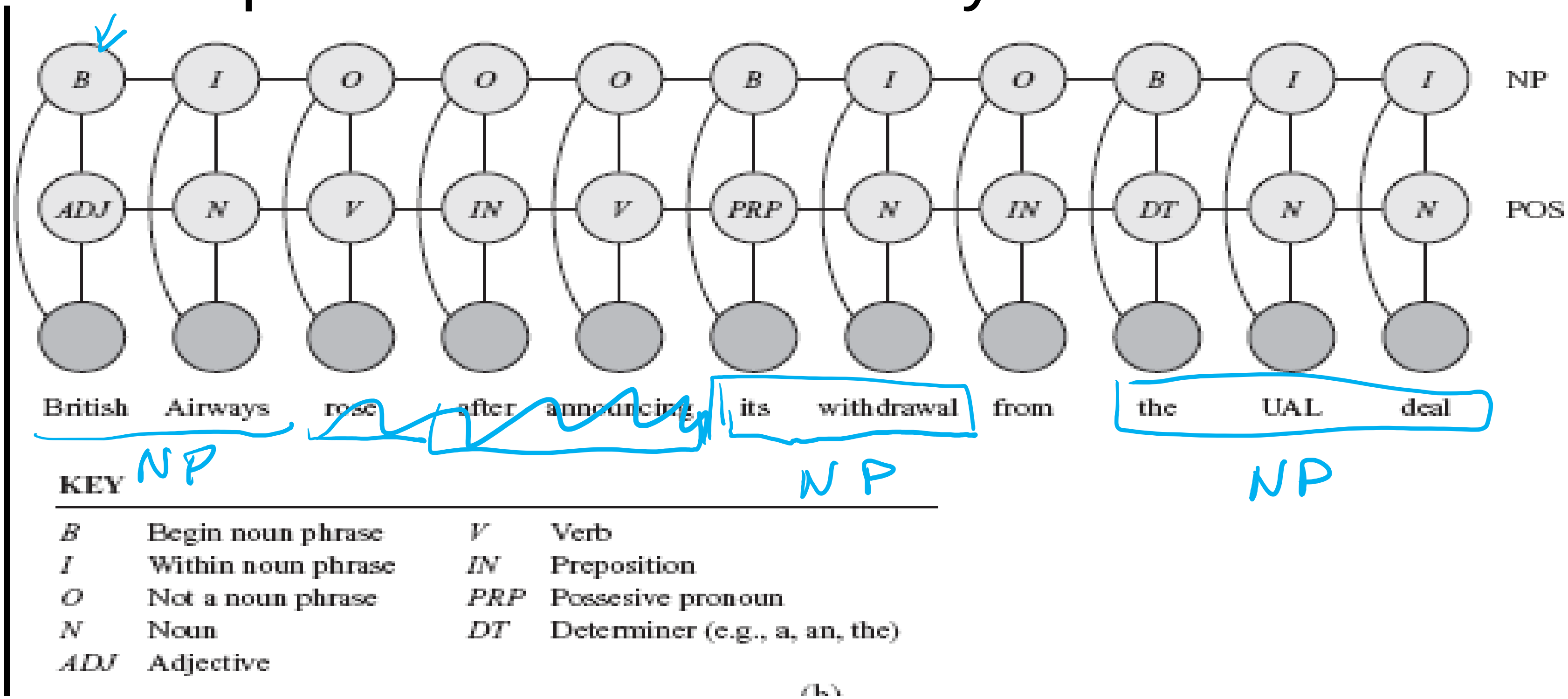
- Linear-chain CRFs can be combined to perform multiple tasks simultaneously



- Performs part-of-speech labeling and noun-phrase segmentation

Coupled linear-chain CRFs

- Linear-chain CRFs can be combined to perform multiple tasks simultaneously

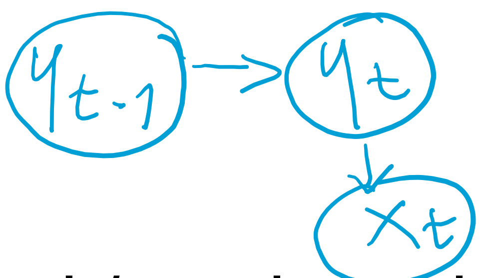


- Performs part-of-speech labeling and noun-phrase segmentation

Inference in CRFs (just intuition)

An HMM can be viewed as a factor graph $\underline{p(\mathrm{y},\mathrm{x})} = \boxed{\prod_t \Psi_t(y_t, y_{t-1}, x_t)}$ where $Z = 1$, and the factors are defined as:

$$\Psi_t(j,i,x) \stackrel{\text{def}}{=} \underline{p(y_t = j|y_{t-1} = i)}\,\underline{p(x_t = x|y_t = j)}. \tag{4.1}$$



Forward / Backward / Smoothing and Viterbi can be rewritten (not trivial!) using these factors

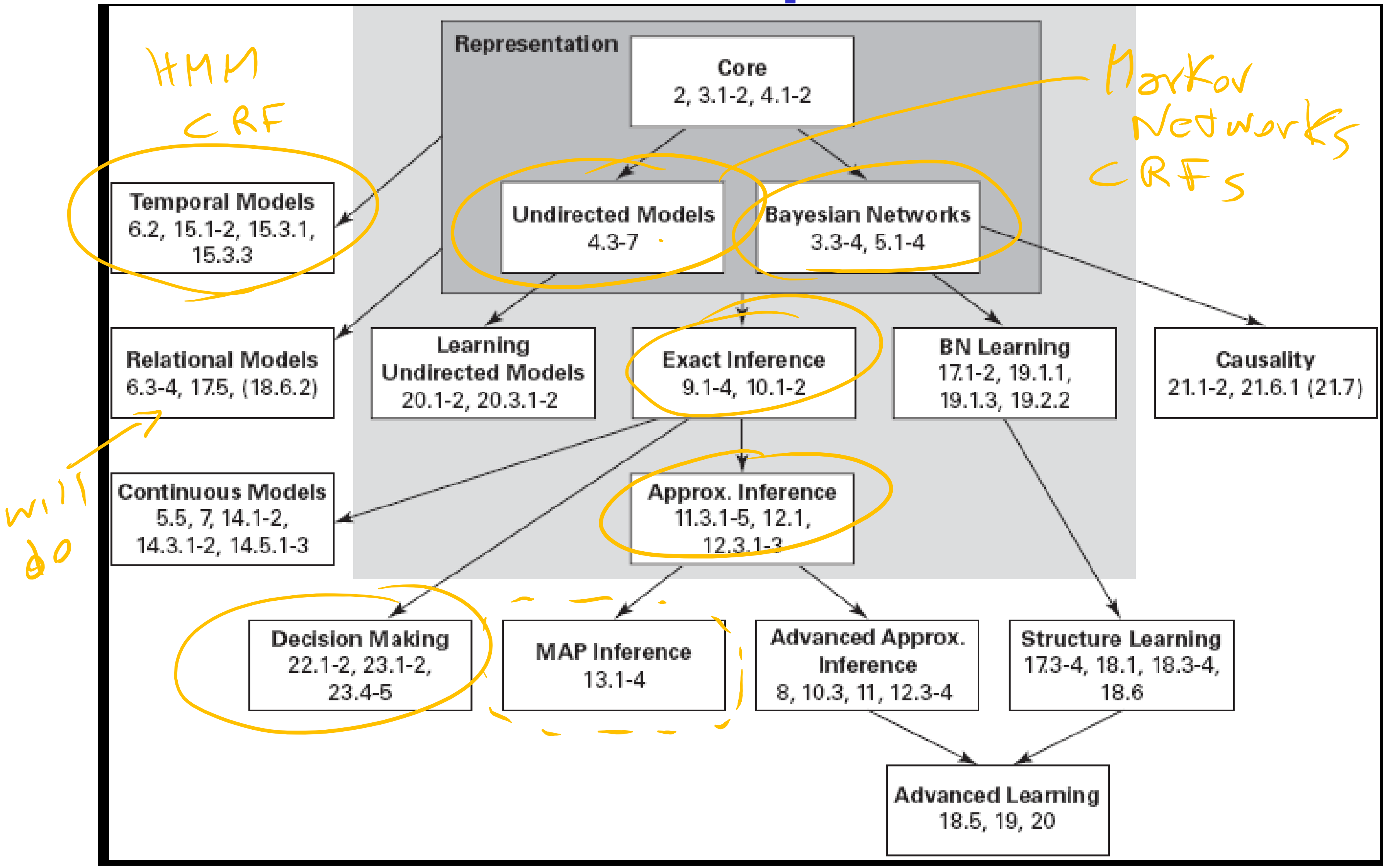
Then you plug in the factors of the CRFs and all the algorithms work fine with CRFs! ☺

CRFs Summary

- Ability to incorporate arbitrary overlapping local and global features
- Graphical structure over Y can depend on the values of the Xs (see slide 24)
- Can perform multiple tasks simultaneously (see slide 26)
- *Standard Inference algorithm* for HMM can be applied
- *Practical Learning algorithms exist*
- Strong baseline on many labeling tasks *(deep learning recently shown to be often better when large training data are available... current research on ensembling them!)*

See MALLET package for CRF implementation

Probabilistic Graphical Models



From "Probabilistic Graphical Models: Principles and Techniques" *D. Koller, N. Friedman* 2009

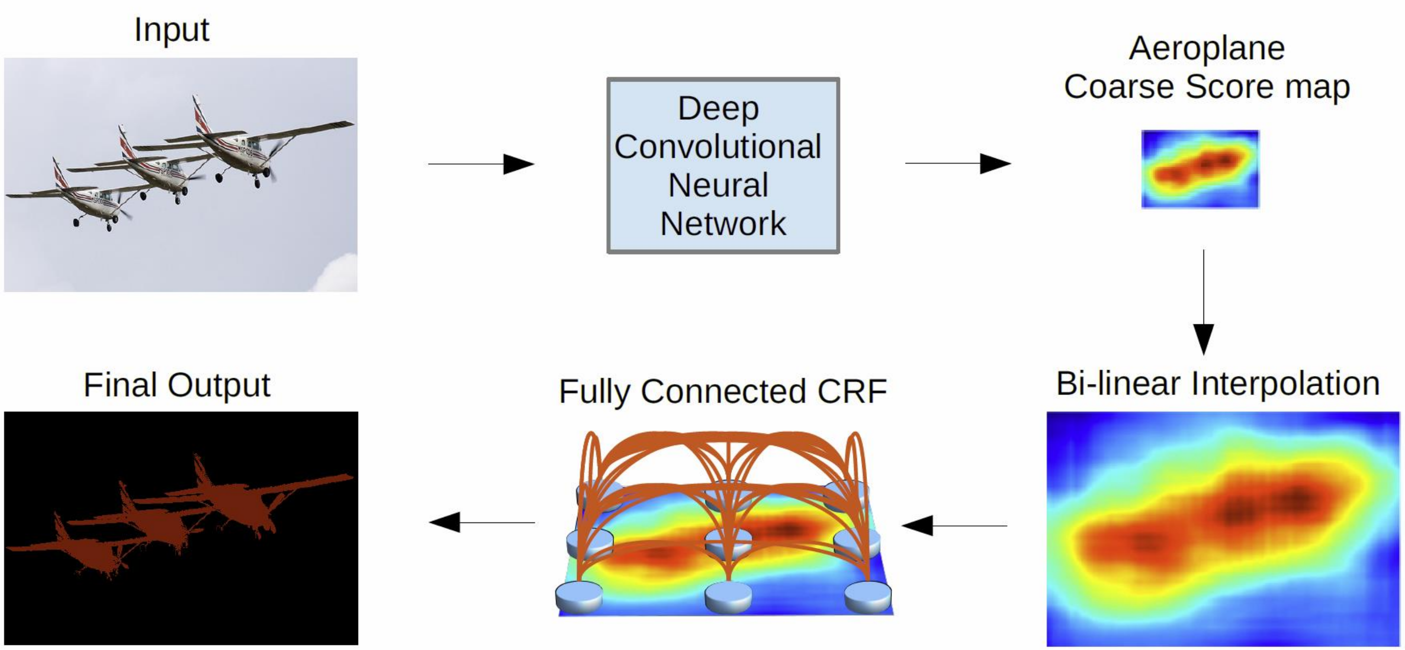
Combining CRFs and Neural Models

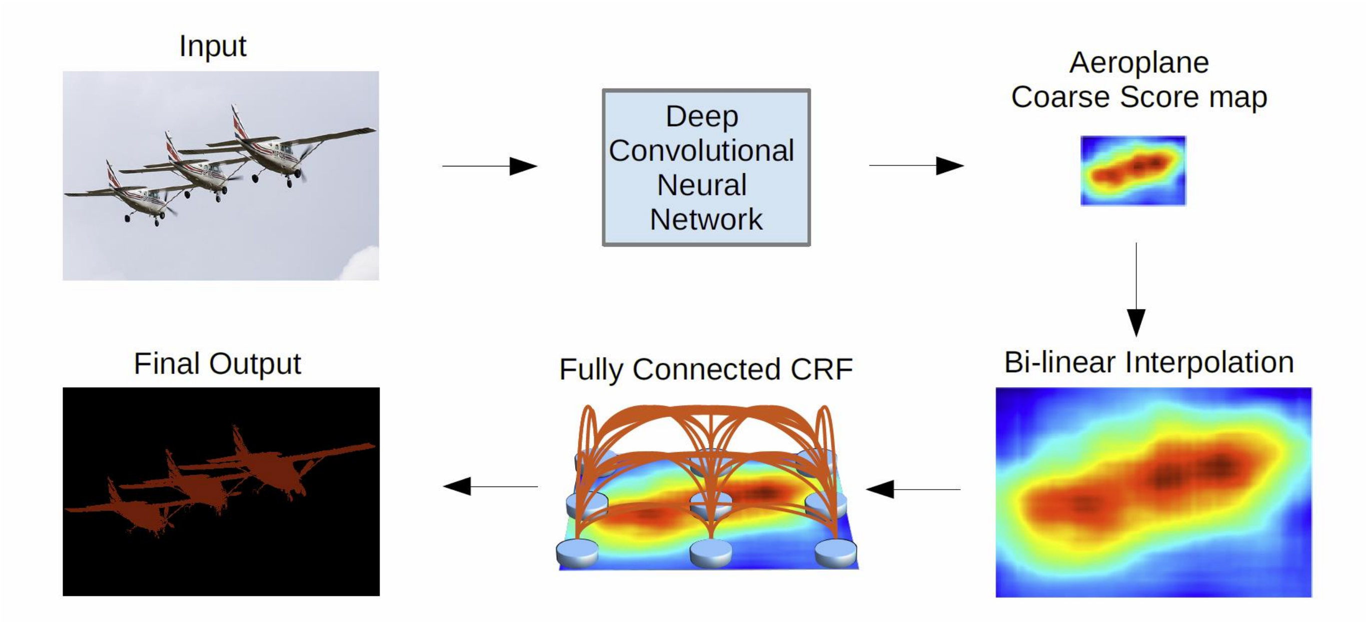
SEMANTIC IMAGE SEGMENTATION WITH DEEP CONVOLUTIONAL NETS AND FULLY CONNECTED CRFS

International Conference on Learning Representations (ICLR), San Diego, California, USA, May 2015.

Liang-Chieh Chen Univ. of California, Los Angeles; George Papandreou Google Inc. ; Iasonas Kokkinos INRIA ; Kevin Murphy Google Inc. ; Alan L. Yuille Univ. of California, Los Angeles

1. Use CNN to generate a rough prediction of segmentation (smooth, blurry heat map)
2. Refine this prediction with a conditional random field (CRF)




Input
Deep Convolutional Neural Network
Aeroplane Coarse Score map
Bi-linear Interpolation
Fully Connected CRF
Final Output

422 big picture

StarAI (statistical relational AI)

Hybrid: Det +Sto

Prob CFG
Prob Relational Models
Markov Logics

Deterministic | **Stochastic**

Query

Logics
First Order Logics
Ontologies

- Full Resolution
- SAT

Belief Nets

Approx. : Gibbs

Markov Chains and HMMs

Forward, Viterbi….
Approx. : Particle Filtering

Undirected Graphical Models
Markov Networks
Conditional Random Fields

Planning

Markov Decision Processes and Partially Observable MDP

- Value Iteration
- Approx. Inference

Reinforcement Learning

Applications of AI

Representation

Reasoning Technique

Learning Goals for today's class

You can:

- Provide general definition for CRF
- Apply CRFs to sequence labeling
- Describe and justify features for CRFs applied to Natural Language processing tasks
- Explain benefits of CRFs

Midterm, Mon, March 8, 4-4:55pm
Check on Piazza for details on format

How to prepare…

- Go to **Office Hours**
- **Learning Goals** (look at the end of the slides for each lecture – complete list has been posted)
- Revise all the **clicker questions** and **practice exercises**
- M**ore practice material** has been posted
- Check questions and answers on Piazza

Next class Wed

- Start Logics
- Revise Logics from 322!

From in class 2017

Conditional Random Fields (CRFs)

- Model $P(Y_1 \,..\, Y_k \mid X_1 .. \, X_n)$
- Special case of Markov Networks where all the X_i are always observed

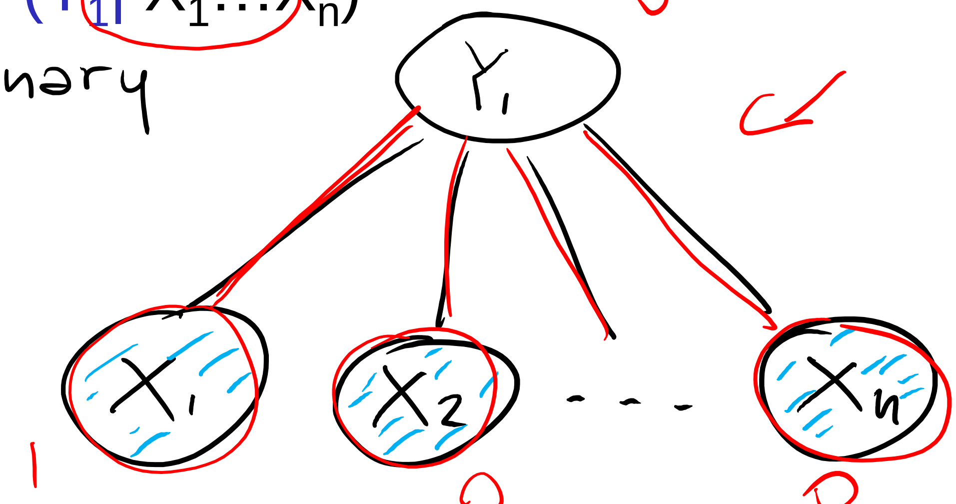
- Simple case $P(Y_1 \mid X_1 \ldots X_n)$

all vars are binary

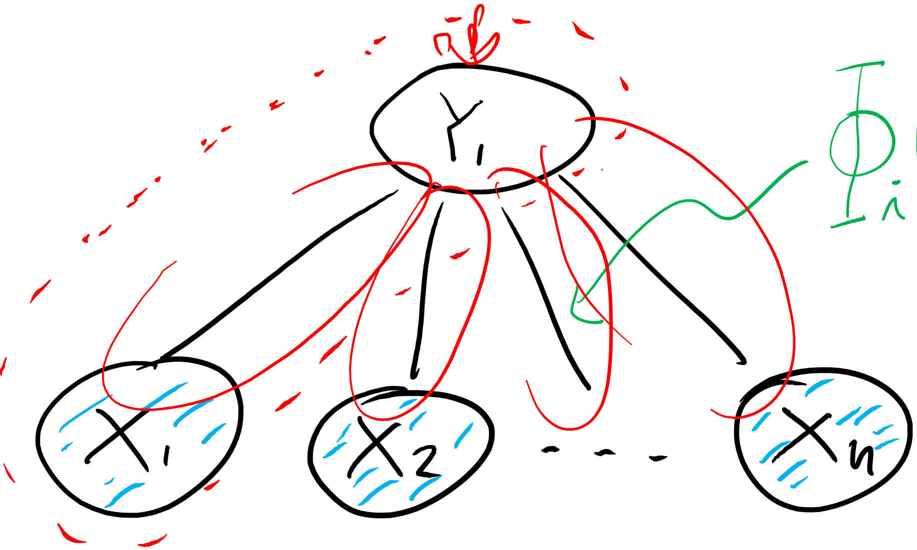
$Y_1 = \{0, 1\}$

$\forall i \;\; X_i = \{0, 1\}$

1/0



What are the Parameters?



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one such factor for each clique

also $\Phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y = 1\}\}$

Example $w_2 = 1.5$ $\Phi_2(X_2, Y_1)$

X_2	Y_1	Φ_2
1	1	$e^{1.5}$
0	1	1
1	0	1
0	0	1

Example $w_0 = .4$ →

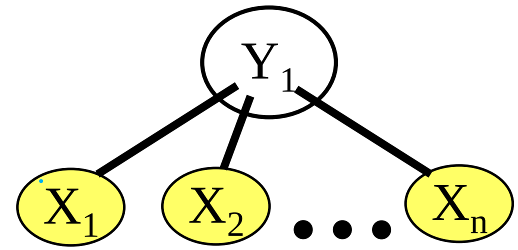
Y_1	Φ_0
0	1
1	$e^{.4}$

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

how strongly $Y_1 = 1$ given that $X_i = 1$

$$\phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y_1 = 1\}\}$$



$$\boxed{P(Y_1 = 1 \mid x_1, \ldots, x_n)} = \frac{P(Y_1 = 1, x_1, \ldots, x_n)}{\boxed{P(x_1, \ldots, x_n)}}$$

$P(Y_1 = 1, \ldots)$

$P(Y_1 = 0, \ldots)$

$$\tilde{P}(Y_1, x_1, \ldots, x_n) = \phi_0(Y_1) * \prod_{i=1}^{n} \phi_i(X_i, Y_1)$$

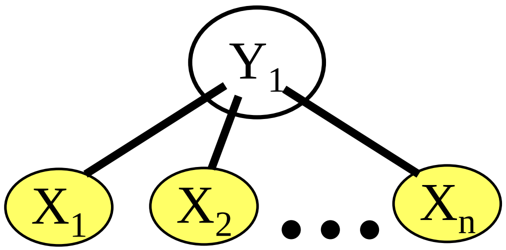
$$\tilde{P}(Y_1 = 0, x_1, \ldots, x_n) = \ ?$$

$$\boxed{\tilde{P}(Y_1 = 1, x_1, \ldots, x_n)} = \ ?$$

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

$$\phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y_1 = 1\}\}$$



$$\tilde{P}(Y_1 = 1, x_1, x_2, \ldots, x_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(x_i, Y_1)$$

example

$$P(Y_1 = 1, X_1 = 0, X_2 = 1, X_3 = 1)$$

$$\boxed{e^{w_0 * 1}} * \boxed{e^{w_1 * 0}} * e^{w_2 * 1} * e^{w_3 * 1}$$

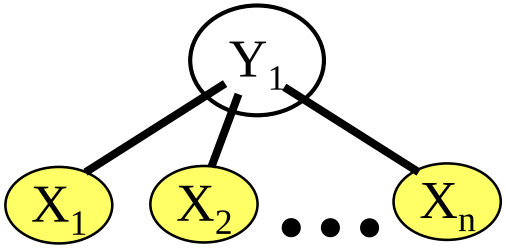
$$e^{w_0} * e^{w_1 * x_1} * e^{w_2 * x_2} * e^{w_3 * x_3} =$$

$$= \boxed{e^{w_0 + \sum w_i x_i}}$$

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \, \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

$$\phi_0(Y_1) = \exp\{w_0 \, \mathbb{1}\{Y_1 = 1\}\}$$



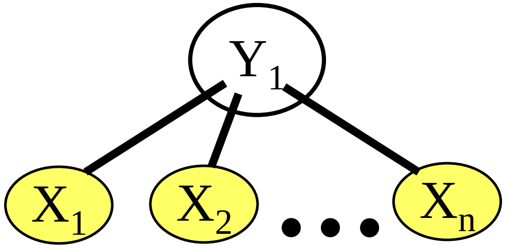
$$\tilde{P}(Y_1 = 0, X_1, X_2, \ldots, X_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(X_i, Y_1)$$

$= 1 \qquad\qquad 1$

Let's derive the probabilities we need

a $\tilde{P}(Y_1 = 1, x_1, \ldots, x_n) = \exp(w_0 + \sum_{i=1}^{n} w_i x_i)$

b $\tilde{P}(Y_1 = 0, x_1, \ldots, x_n) = 1$



$$P(Y_1 = 1 \mid x_1, \ldots, x_n) = \frac{\tilde{P}(Y_1 = 1, x_1, \ldots, x_n)}{\tilde{P}(x_1, \ldots, x_n)}$$

sum of a and b

$$= \frac{\exp(w_0 + \sum w_i x_i)}{1 + \exp(w_0 + \sum w_i x_i)}$$

a

a + b

Continue.....

$z = w_0 + \sum w_i x_i$

$$P(Y_1 = 1 \mid X_1 \ldots X_n) = \frac{e^{w_0 + \sum w_i x_i}}{1 + e^{w_0 + \sum w_i x_i}}$$

$$1 - \frac{1}{e^{-z} + 1}$$

$$= \frac{e^{z}}{1 + e^{z}} \; \frac{e^{-z}}{e^{-z}} = \frac{1}{e^{-z} + 1}$$

$$P(Y_1 \mid X_1 \ldots X_n) = \left\{ \frac{1}{e^{-z} + 1} \;\middle|\; \frac{e^{-z}}{e^{-z} + 1} \right\}$$

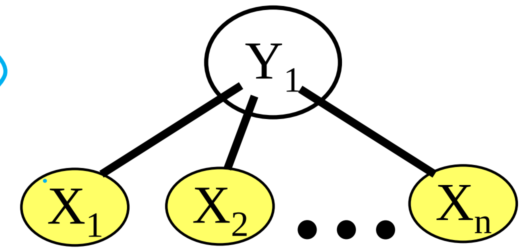
END in class 2017

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

how strongly $Y_1 = 1$ given that $X_i = 1$

$$\phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y_1 = 1\}\}$$



$$P(\overset{A}{Y_1} \mid \overset{B}{x_1, \ldots, x_n}) = \frac{\tilde{P}(Y, x_1 \ldots x_n)}{\tilde{P}(x_1 \ldots x_n)}$$

$$\boxed{\tilde{\tilde{P}}(Y_1, x_1, \ldots, x_n)} = \phi_0(Y_1) * \prod_{i=1}^{n} \phi_i(X_i, Y_1)$$

$$\tilde{\tilde{P}}(Y_1 = 0, x_1, \ldots, x_n) = 1$$

$$\tilde{\tilde{P}}(Y_1 = 1, x_1, \ldots, x_n) = e^{w_0 + \sum w_i x_i}$$

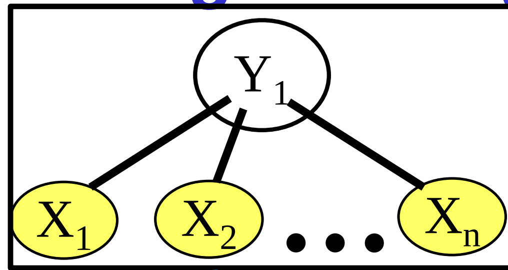
Continue…..

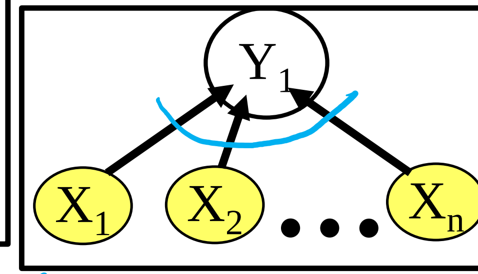
$$P(Y_1=1 \mid X_1 \ldots X_n) = \frac{e^{w_0+\sum w_i X_i}}{1 + e^{w_0+\sum w_i X_i}}$$

$$= \frac{e^{z}}{1+e^{z}} \, \frac{e^{-z}}{e^{-z}} = \frac{1}{e^{-z}+1}$$

$$P(Y_1 \mid X_1 \ldots X_n) = \left\{ \frac{1}{e^{-z}+1} \, , \, \frac{e^{-z}}{e^{-z}+1} \right\}$$

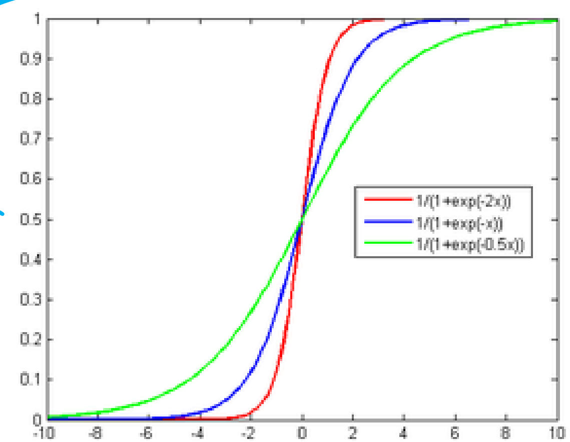
Sigmoid Function used in Logistic Regression



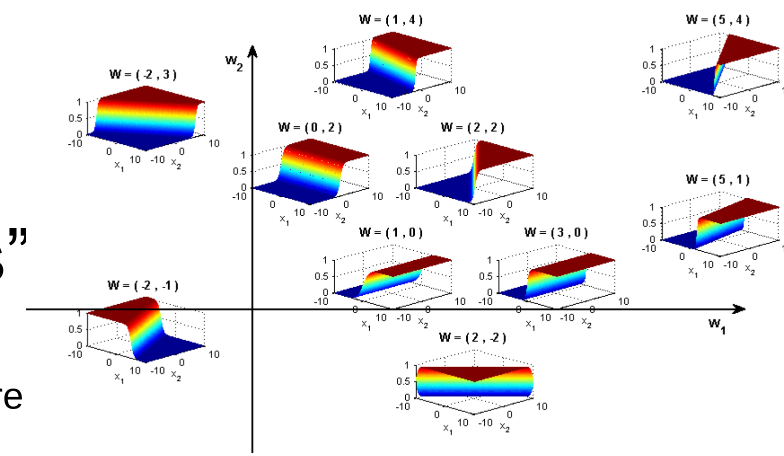


- Great practical interest
- Number of param w_i is linear instead of exponential in the number of parents

$\frac{1}{1+e^{-x}}$

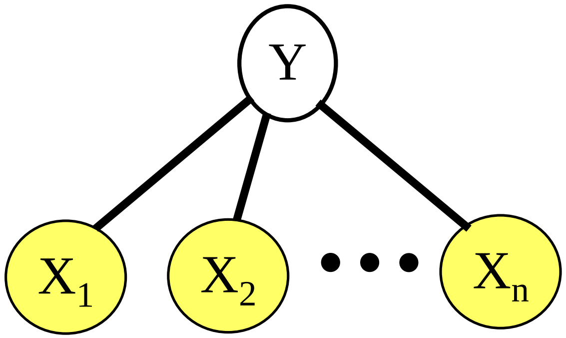


- Natural model for many real-world applications
- Naturally aggregates the influence of different "parents"



Logistic Regression as a Markov Net (CRF)

Logistic regression is a simple Markov Net (a CRF) *aka* **naïve markov model**



- But only models the **conditional distribution**, P(Y | $\boldsymbol{X}$) and not the full joint P(X,Y)