Uniformed Search (cont.)

Computer Science cpsc322, Lecture 6

(Textbook finish 3.5)

Sept, 16, 2013
Lecture Overview

• Recap DFS vs BFS

• Uninformed Iterative Deepening (IDS)

• Search with Costs
Search Strategies are different with respect to how they:

A. Check what node on a path is the goal
B. Initialize the frontier
C. Add/remove paths from the frontier
D. Check if a state is a goal
Recap: Graph Search Algorithm

Input: a graph, a start node, Boolean procedure $goal(n)$ that tests if $n$ is a goal node

$frontier := [<s> : s$ is a start node$]$;

While $frontier$ is not empty:

select and remove path $<n_0, ..., n_k>$ from $frontier$;

If $goal(n_k)$

return $<n_0, ..., n_k>$;

For every neighbor $n$ of $n_k$

add $<n_0, ..., n_k, n>$ to $frontier$;

end

In what aspects DFS and BFS differ when we look at the generic graph search algorithm?
When to use BFS vs. DFS?

- The search graph has cycles or is infinite
  - BFS
  - DFS

- We need the shortest path to a solution
  - BFS
  - DFS

- There are only solutions at great depth
  - BFS
  - DFS

- There are some solutions at shallow depth
  - BFS
  - DFS

- Memory is limited
  - BFS
  - DFS
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Iterative Deepening (sec 3.6.3)

How can we achieve an acceptable (linear) space complexity maintaining completeness and optimality?

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<th>Complete</th>
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<th>Time</th>
<th>Space</th>
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<tr>
<td>DFS</td>
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<td>BFS</td>
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Key Idea: let’s re-compute elements of the frontier rather than saving them.
Iterative Deepening in Essence

• Look with DFS for solutions at depth 1, then 2, then 3, etc.

• If a solution cannot be found at depth $D$, look for a solution at depth $D + 1$.

• You need a depth-bounded depth-first searcher.

• Given a bound B you simply assume that paths of length B cannot be expanded....
depth = 1

depth = 2

depth = 3
(Time) Complexity of Iterative Deepening

Complexity of solution at depth $m$ with branching factor $b$

Total # of paths at that level

#times created by BFS (or DFS)

#times created by IDS

Total # of paths created by IDS
**(Time) Complexity of Iterative Deepening**

Complexity of solution at depth $m$ with branching factor $b$

Total # of paths generated

$$b^m + 2b^{m-1} + 3b^{m-2} + \ldots + mb = b^m \left(1 + 2b^{-1} + 3b^{-2} + \ldots + m b^{1-m}\right) \leq b^m \left(\sum_{i=1}^{\infty} ib^{1-i}\right) = b^m \left(\frac{b}{b-1}\right)^2 = O(b^m)$$

Calculations for specific values of $b$:

- $b = 2$: $\frac{9}{4} = 2.25$
- $b = 3$: $\frac{16}{4} = 4$
- $b = 4$: $\frac{16}{4} < 2$
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Example: Romania
Search with Costs

Sometimes there are costs associated with arcs.

**Definition (cost of a path)**
The cost of a path is the sum of the costs of its arcs:

\[
\text{cost}\left(\langle n_0, \ldots, n_k \rangle\right) = \sum_{i=1}^{k} \text{cost}\left(\langle n_{i-1}, n_i \rangle\right)
\]

In this setting we often don't just want to find just any solution
- we usually want to find the solution that minimizes cost

**Definition (optimal algorithm)**
A search algorithm is optimal if, when it returns a solution, it is the one with minimal.
Lowest-Cost-First Search

- At each stage, lowest-cost-first search selects a path on the frontier with **lowest cost**.
  - The frontier is a **priority queue ordered by path cost**
  - We say ``a path'' because there may be ties

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**Example of one step for LCFS:**

- the frontier is \([\langle p_2, 5 \rangle, \langle p_3, 7 \rangle, \langle p_1, 11 \rangle, ]\)
- \(p_2\) is the lowest-cost node in the frontier
- “neighbors” of \(p_2\) are \(\{\langle p_9, 10 \rangle, \langle p_{10}, 15 \rangle\}\)

What happens?

- \(p_2\) is selected, and tested for being a goal.
- Neighbors of \(p_2\) are inserted into the frontier
- Thus, the frontier is now \([\langle p_3, 7 \rangle, \langle p_9, 10 \rangle, \langle p_1, 11 \rangle, \langle p_{10}, 15 \rangle]\).
- \(? \ p_3 \ ?\) is selected next.
- Etc. etc.
When arc costs are equal LCFS is equivalent to:

A. DFS  
B. BFS  
C. IDS  
D. None of the Above
Analysis of Lowest-Cost Search (1)

- Is LCFS complete?
  - not in general: a cycle with zero or negative arc costs could be followed forever.
  - yes, as long as arc costs are strictly positive.

- Is LCFS optimal?
  - Not in general. Why not?
  - Arc costs could be negative: a path that initially looks high-cost could end up getting a ``refund''.
  - However, LCFS is optimal if arc costs are guaranteed to be non-negative.
Analysis of Lowest-Cost Search

• What is the time complexity, if the maximum path length is $m$ and the maximum branching factor is $b$?
  • The time complexity is $O(b^m)$: must examine every node in the tree.
  • Knowing costs doesn't help here.

• What is the space complexity?
  • Space complexity is $O(b^m)$: we must store the whole frontier in memory.
Learning Goals for Search (up to today)

- Apply basic properties of search algorithms: completeness, optimality, time and space complexity of search algorithms.

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Learning Goals for Search (cont’)
(up to today)

- Select the most appropriate search algorithms for specific problems.
  - BFS vs DFS vs IDS vs BidirS-
  - LCFS vs BFS
  - A* vs B&B vs IDA* vs MBA*

- Define/read/write/trace/debug different search algorithms
  - With / Without cost
  - Informed / Uninformed
Beyond uninformed search….

What information we could use to better select paths from the frontier

A. an estimate of the distance from the last node on the path to the goal
B. an estimate of the distance from the start state to the goal
C. an estimate of the cost of the path
D. None of the above
Next Class

- Start Heuristic Search
  (textbook.: start 3.6)