CS303: Recap

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Roundoff errors: Goals

- Get familiar with the causes for roundoff error generation
- Understand roundoff error propagation
- Understand rough, random-like appearance of roundoff error and cancellation error.

Left out material from notes

- Sections 2.2, 2.3 and 2.4.

Polynomial interpolation: Goals

- Get throroughly familiar with a most important building block for numerical algorithms
- Study approximations in the form $v(x) = \sum_{j=0}^{n} c_j \phi_j(x)$
- Understand polynomial bases:
  - Monomial (ad: simple)
  - Lagrange (ad: stable, $c_j = y_j$)
  - Newton (ad: adaptive)
- Obtain approximation error bound for polynomial interpolation
- Derive global, high degree polynomial interpolation: Chebyshev
- Find out how to interpolate also derivative values

Left out material from notes

- Most of the theory concerning divided differences.
- More general cases of osculating interpolation

Piecewise polynomial interpolation: Goals

- Derive and understand most common interpolation schemes in practical use:
  - Piecewise linear (broken-line)
  - Piecewise Hermite cubic
  - Cubic spline
- Extend to parametric curves and see application in computer-aided geometric design
- Derive basis functions with local support
  - Hat functions
  - Hermite basis functions
  - B-splines

Left out material from notes

- Deriving the cubic spline (Section 11.3)
- Most of the material on basis functions (Section 11.4)
- Multidimensional interpolation (Section 11.6)

Best approximation: Goals

- Get acquainted with function norms
- Obtain algorithm for best least squares approximation
- Encounter families of orthogonal polynomials:
  - Legendre polynomials
  - Chebyshev polynomials

Left out material from notes

- Sections 12.3 and 12.4.

Fourier transform: Goals

- Get acquainted with the transform, the inverse transform and their applications
- Obtain algorithm for discrete Fourier transform (DFT) and examine its properties
- Understand advantages and limitations of this approximation
- Derive the fast Fourier transform (FFT)
- Discrete cosine transform (DCT)

Left out material from notes

- Deblurring example in Section 13.3.

Numerical differentiation: Goals

- Derive basic, useful discretization formulas for 1st and 2nd derivatives
- Get a grasp of the art of using Taylor expansions
- Understand limitations in terms of roundoff error accumulation and propagation, and the amplification of measurement errors upon numerical differentiaion

Left out material from notes

- Sections 14.2, 14.3 and 14.5.

Numerical integration: Goals

- Understand the principles of numerical integration methods
- Introduce concepts that are important beyond the scope of this chapter (a primer for DEs)
- Derive useful methods, basic and composite, and study their performance
  - Trapezoidal method
  - Simpson method
  - Gaussian quadrature (including midpoint)

Left out material from notes

- Sections 15.4, 15.5 and 15.6.

Differential equations: Goals

- Get an idea of problems and methods for various differential equations, concentrating on initial value ODEs
- Introduce concepts: order, local truncation error, global error, absolute stability, etc.
- Derive useful methods and study their performance
  - Forward Euler
  - Backward Euler
  - Trapezoidal (implicit)
  - Explicit Runge-Kutta: 2-stage 2nd order, 4-stage 4th order
  - RK pair
- Understand error control and step size selection
- Study methods for stiff problems and their implementation

Left out material from notes

- Sections 16.4, 16.7 and 16.8.
- The more challenging parts of Sections 16.5 and 16.6.