Department of Computer Science
CPSC 303 Midterm Examination II (U. Ascher)

Nov. 17, 2004. NAME:________________________________

Number of pages: 4 Signature:________________________________

Time: 50 minutes STD. NUM: ________________________________

You are permitted a calculator and one double-sided $8\frac{1}{2} \times 11$ sheet of notes to assist you in answering the questions.

For the short answer questions, be as concise as possible. The **maximum** total of the marks is 100: *use your time wisely.* Show all your work.

Good Luck!

Q1	Q2	Q3	Q4	TOTAL
30	30	30	30	100

1. **[30 marks]**

The following census data for the population (in millions) of a certain country were obtained:

Year	1980	1990	2000
Population	228	250	282

a) Use polynomial interpolation to predict the population in the year 2005 based on these data. Will the projected population exceed 300 million?

b) Same question using only the data of the last two years, 1990 and 2000.

2. **[30 marks]**

The composite trapezoidal method has the error expression $E(f) = -\frac{f''(\eta)}{12}(b-a)h^2$, while the composite Simpson boasts the expression $E(f) = -\frac{f''''(\zeta)}{180}(b-a)h^4$.

Let r be the minimal number of subintervals required in order to approximate $\int_0^1 e^{-x}dx$ to within an absolute error tolerance of 10^{-8}:

a) Using the composite trapezoidal method, is r below or above 1000?

b) Using the composite Simpson method, is r below or above 35?

Note: No marks will be awarded for successful guesses: show your work and reasoning!

3. **[30 marks]**

Recall that the Chebyshev polynomials satisfy the recursion formula

$$\begin{aligned} T_0(x) &= 1, \\ T_1(x) &= x, \\ T_{j+1}(x) &= 2xT_j(x) - T_{j-1}(x). \end{aligned}$$

a) Show that $T_n(x)$ can be written as

$$T_n(x) = 2^{n-1}(x - \zeta_1)(x - \zeta_2)\cdots(x - \zeta_n),$$

where ζ_i are the n roots of T_n.

b) Jane works for for a famous bioinformatics company. Last year she was required by Management to represent an important but complicated formula, $g(x)$, defined on the interval $[-1, 1]$, by a polynomial of degree $n + 1$. She did so, and called the result $f(x) = p_{n+1}(x)$.

Last week Management has decided that they really needed a polynomial of degree n, not $n + 1$, to represent g. Alas, the original g had been lost by this time and all that was left was $f(x)$. Therefore, Jane is looking for the polynomial of degree n which is closest (in the maximum norm) to f on the interval $[-1, 1]$. Please help her find it.

4. **[30 marks]** We have encountered the following three formulae for approximating the first derivative of $f(x)$ at $x = x_0$:

$$\begin{aligned} D_1 &= \frac{1}{h}\left(f(x_0+h) - f(x_0)\right), \\ D_2 &= \frac{1}{2h}\left(f(x_0+h) - f(x_0-h)\right), \\ D_3 &= \frac{1}{12h}\left(f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right). \end{aligned}$$

The corresponding truncation errors are

$$|e(h)| = \frac{h}{2}f''(\xi_1), \quad \frac{h^2}{6}f'''(\xi_2) \quad \text{and} \quad \frac{h^4}{30}f^{(v)}(\xi_3),$$

respectively.

a) Using the above information find $D_1(h)$, $D_2(h)$ and $D_3(h)$ for $f(x) = x^3$ at $x_0 = 1$. (Ignore roundoff errors.)

b) Find the corresponding truncation errors.

Hint: You may find the following formula useful:

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3.$$