Department of Computer Science
CPSC 303 Midterm Examination I (U. Ascher)

Oct. 13, 2004. NAME:______________________________

Number of pages: 4 Signature:______________________________

Time: 50 minutes STD. NUM: ______________________________

You are permitted a calculator and one double-sided $8\frac{1}{2} \times 11$ sheet of notes to assist you in answering the questions.

For the short answer questions, be as concise as possible. The weight of each question is given in parentheses. The total number of marks is 100 (approximately 2 marks/min). Show all your work.

Good Luck!

Q1	Q2	Q3	Q4	Q5	TOTAL
15	15	20	20	30	100

1. [**15 marks**] Please circle **TRUE** or **FALSE** as appropriate.

   (a) Rounding is better than simple chopping as a method for storing real numbers in a floating point system, not only because the error is half as large but also because statistically the accumulated error in a floating point calculation is significantly smaller.

   TRUE / FALSE

   (b) The evaluation of a *parametric* interpolating polynomial of degree n in Newton form costs $\mathcal{O}(n^2)$ flops

   TRUE / FALSE

   (c) Let $f(x) = 2 + e^x$ and $p_1(x)$ its interpolating polynomial at $x_0 = 0$ and $x_1 = 1$. Then the relative interpolation error on $[0, 1]$ is larger than the corresponding absolute error.

   TRUE / FALSE

(d) In a normalized floating point number representation the first digit in the fraction (mantissa) cannot be zero.

TRUE / FALSE

(e) The error in the Lagrange polynomial interpolation form is smaller than the corresponding one in Newton form when the data values equal 1 at the first interpolation point and 0 at all other interpolation points.

TRUE / FALSE

2. [**15 marks**]

The roots of the quadratic equation

$$x^2 - 2bx + c = 0$$

with $b^2 > c$ are given by

$$x_{1,2} = b \pm \sqrt{b^2 - c}.$$

Note $x_1 x_2 = c$.

The following MATLAB scripts calculate these roots using two different algorithms:

(a)
```
x1 = b + sqrt(b^2-c);
x2 = b - sqrt(b^2-c);
```

(b)
```
if b > 0
  x1 = b + sqrt(b^2-c);
  x2 = c / x1;
else
  x2 = b - sqrt(b^2-c);
  x1 = c / x2;
end
```

Which algorithm gives a more accurate result when $|b| \gg \sqrt{|c|}$? Justify in one short sentence. *Note: No justification, or two justifications, produce no marks. Also, assume that no overflow occurs in any of these operations.*

3. **[20 marks]** A secret formula for eternal youth, $f(x)$, was discovered by Dr. Quack, who has been working in our biotech company. However, Dr. Quack has disappeared and is rumored to be negotiating with a rival organization.

   From the notes that Dr. Quack left behind in his hasty exit it is clear that $f(0) = 1$, $f'(0) = 3$, and that $f[x_0, x_1, x_2] = -0.5$ for *any* three points x_0, x_1, x_2.

   Find $f(x)$.

4. **[20 marks]**

   Given function values $f(t_0), f(t_1), \ldots, f(t_q)$, as well as those of $f'(t_0)$ and $f'(t_q)$, for some $q \geq 2$, it is possible to construct the complete interpolating cubic spline.

   Suppose that we were instead to approximate $f'(t_i)$ by $f[t_{i-1}, t_{i+1}]$, for $i = 1, 2, \ldots, q-1$, and then use these values (as well as the given ones) to construct a Hermite piecewise cubic interpolant.

   Give one advantage and one disadvantage of this procedure over a complete cubic spline interpolation.

   Note: If more than one advantage or one disadvantage is given then the least correct will be graded.

5. **[30 marks]**

Suppose that a given function $f \in C^4[0,1]$ is to be interpolated from values of f or f' at $2N$ points in $[0,1]$, where $N \geq 5$ is an integer. Three options are considered:

(a) Sample f at $2N$ equidistant points (including 0 and 1) and construct the *natural cubic spline*.

(b) Sample f at $2(N-1)$ equidistant points (including 0 and 1), and in addition $f'(0)$ and $f'(1)$, and construct the *clamped*, or *complete cubic spline*.

(c) Sample both f and f' at N equidistant points (including 0 and 1) and construct the *Hermite piecewise cubic*.

Questions:

(a) knowing nothing else about f, which of these three interpolants would you expect to be the *most* accurate? Justify in one brief sentence.

(b) knowing nothing else about f, which of these three interpolants would you expect to be the *least* accurate? Justify in one brief sentence.