Department of Computer Science
CPSC 303 Midterm Examination (U. Ascher)

Oct. 23, 2002. NAME:____________________

Number of pages: 6 Signature:____________________

Time: 50 minutes STD. NUM: ____________________

You are permitted a calculator and one double-sided $8\frac{1}{2} \times 11$ sheet of handwritten notes to assist you in answering the questions.

For the short answer questions, be as concise as possible. The weight of each question is given in parentheses. The total number of marks is 100 (approximately 2 marks/min). If you run out of space for a question, use the reverse side. Show all your work.

Good Luck!

Q1	Q2	Q3	Q4	TOTAL
20	30	20	30	100

1. **[20 marks]** The roots of the quadratic equation

$$x^2 - 2bx + c = 0$$

with $b^2 > c$ are given by

$$x_{1,2} = b \pm \sqrt{b^2 - c}.$$

Note $x_1 x_2 = c$.

The following MATLAB scripts calculate these roots using two different algorithms:

(a)
```
x1 = b + sqrt(b^2-c);
x2 = b - sqrt(b^2-c);
```

(b)
```
if b > 0
  x1 = b + sqrt(b^2-c);
  x2 = c / x1;
else
  x2 = b - sqrt(b^2-c);
  x1 = c / x2;
end
```

Which algorithm gives a more accurate result in general? – circle one of:

1. Algorithm (i)   2. Algorithm (ii)   3. Both algorithms produce the same result

Justify your choice in one short sentence. *Note: No justification, or two justifications, produce no marks. Also, assume that no overflow occurs in any of these operations.*

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2. A popular technique arising in methods for minimizing functions in several variables involves a *weak line search*, where an approximate minimum x^* is found for a function in one variable, $f(x)$, for which the values of $f(0)$, $f'(0)$ and $f(1)$ are given. The function $f(x)$ is defined for all nonnegative x, has a continuous second derivative, and satisfies $f(0) < f(1)$ and $f'(0) < 0$. We then interpolate the given values by a quadratic polynomial and set x^* as the minimum of the interpolant.

(a) **[30 marks]** Find x^* for the values $f(0) = 1,\ f'(0) = -1,\ f(1) = 2.$

(b) **Bonus** (*do not attempt this part unless you have extra time left*):
Show that the quadratic interpolant has a unique minimum satisfying $0 < x^* < 1$. Can you show the same for the function f itself?

3. **[20 marks]**

Given function values $f(t_0), f(t_1), \ldots, f(t_q)$, as well as those of $f'(t_0)$ and $f'(t_q)$, for some $q \geq 2$, it is possible to construct the complete interpolating cubic spline.

Suppose that we were instead to approximate $f'(t_i)$ by $f[t_{i-1}, t_{i+1}]$, for $i = 1, 2, \ldots, q-1$, and then use these values (as well as the given ones) to construct a Hermite piecewise cubic interpolant.

Give one advantage and one disadvantage of this procedure over a complete cubic spline interpolation.

Note: If more than one advantage or one disadvantage is given then the least correct will be graded.

4. Often in practice, an approximation of the form

$$u(x) = \gamma_0 e^{\gamma_1 x}$$

is sought for a data fitting problem, where γ_0 and γ_1 are constants.

Assume given data $(x_0, z_1), (x_0, z_1), \ldots, (x_m, z_m)$, where $z_i > 0,\ i = 0, 1, \ldots, m$, and $m > 0$.

(a) [**10 marks**] Explain in one brief sentence why the techniques introduced in class cannot be directly applied to find this $u(x)$.

(b) [**20 marks**] Considering instead

$$v(x) = \ln u(x) = (\ln \gamma_0) + \gamma_1 x$$

it makes sense to define $y_i = \ln z_i,\ i = 0, 1, \ldots, m$, and then find coefficients c_0 and c_1 such that $v(x) = c_0 + c_1 x$ is the best least squares fit for the data

$$(x_0, y_0), (x_1, y_1), \ldots, (x_m, y_m).$$

Using this method, find $u(x)$ for the data

i	0	1	2
x_i	0.0	1.0	2.0
z_i	$e^{0.1}$	$e^{0.9}$	e^{2}