CPSC 303, Fall Term, 2010
Assignment 5, due Friday November 26th

Each question is worth 30 marks, for a maximum total of 100.

Please show all your work: e-mail your MATLAB programs to `cs303@ugrad.cs.ubc.ca`, but provide a hardcopy of the rest of the assignment. When e-mailing your programs, include your name and student ID in the message's title.

Do not e-mail a complete assignment.

1. Consider the ODE
$$\frac{dy}{dt} = f(t, y), \qquad 0 \le t \le b,$$
where $b \gg 1$.
   (a) Apply the *stretching* transformation $t = \tau b$ to obtain the equivalent ODE
   $$\frac{dy}{d\tau} = b\ f(\tau b, y), \qquad 0 \le \tau \le 1.$$
   (Strictly speaking, y in these two ODEs is not quite the same function. Rather, it stands in each case for the unknown function.)
   (b) Show that applying the forward Euler method to the ODE in t with step size $h = \Delta t$ is equivalent to applying the same method to the ODE in τ with step size $\Delta\tau$ satisfying $\Delta t = b\Delta\tau$. In other words, the same stretching transformation can be equivalently applied to the discretized problem.

2. To draw a circle of radius r on a graphics screen, one may proceed to evaluate pairs of values $x = r\cos\theta$, $y = r\sin\theta$ for a succession of values θ. But this is computationally expensive. A cheaper method may be obtained by considering the ODE
$$\dot{x} = -y, \qquad x(0) = r,$$
$$\dot{y} = x, \qquad y(0) = 0,$$

where $\dot{x} = \frac{dx}{d\theta}$, and approximating this using a simple discretization method. However, care must be taken so as to ensure that the obtained approximate solution looks right, i.e., that the approximate curve closes rather than spirals.

Carry out this integration using a uniform step size $h = .02$ for $0 \leq \theta \leq 120$, applying forward Euler, backward Euler, and the implicit trapezoidal method. Determine if the solution spirals in, spirals out, or forms an approximate circle as desired. Explain the observed results. [Hint: This has to do with a certain invariant function of x and y, rather than with the accuracy order of the methods.]

3. The 3-stage RK method given by the tableau

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$$

is based, like the classical RK method, on the Simpson quadrature rule.

Implement this method and add a corresponding column to Table 16.2 in the text. What is the observed order of the method?

4. The ODE $u'' = \frac{1}{t}u' - 4t^2 u$ has the solution $u(t) = \sin(t^2) + \cos(t^2)$.

   (a) Plot the exact solution over the interval $[1, 20]$.

   (b) Integrate this problem on the interval $[1, 20]$ using `ode45` with default tolerances (specify initial values $u(1)$ and $u'(1)$ based on the given exact solution), and record the absolute error in u at $t = 20$ and the number of steps required to get there.

   Then use our `rk4` with a uniform step size $h = .01$ for the same purpose. Compare errors and step counts, and comment.

   (c) Now use MATLAB's `odeset` to change the relative error tolerance in `ode45` to 1.e-6. Repeat the run as above. Repeat also the `rk4` run, this time with a uniform step size $h = .002$. Compare errors and step counts, and comment further. Do not become cynical.