CPSC 303, Fall Term, 2010
Assignment 3, due Monday November 1st

Please show all your work: e-mail your MATLAB programs to `cs303@ugrad.cs.ubc.ca`, but provide a hardcopy of the rest of the assignment. When e-mailing your programs, include your name and student ID in the message's title.

Do not e-mail a complete assignment.

1. The Legendre polynomials satisfy

$$\int_{-1}^{1} \phi_j(x)\phi_k(x)dx = \begin{cases} 0 & j \neq k \\ \frac{2}{2j+1} & j = k. \end{cases}$$

   Suppose that the best fit problem is given on the interval $[a, b]$.

   Show that with the transformation $t = \frac{1}{2}[(b-a)x + (a+b)]$ and a slight change of notation, we have

$$\int_{a}^{b} \phi_j(t)\phi_k(t)dt = \begin{cases} 0 & j \neq k \\ \frac{b-a}{2j+1} & j = k. \end{cases}$$

2. Redo Example 12.1 in the notes, reconstructing Figure 12.1, using an orthogonal polynomial base.

3. Consider using a DFT to interpolate the function $f(x) = \log(x+1)$ on the interval $[0, 2\pi]$ as in the examples of Section 13.2.

   (a) Construct and plot the interpolant on $[0, 2\pi]$ for $l = 16$ and $l = 32$. Explain why the results look unsatisfactory.

   (b) Consider an even extension of $f(x)$, defining

$$g(t) = \begin{cases} f(t) & 0 \leq t < 2\pi \\ f(4\pi - t) & 2\pi \leq t < 4\pi \end{cases}.$$

Apply DFT interpolation to $g(t)$ and plot the results on $[0, 2\pi]$. Find maximum errors for $l = 16$ and $l = 32$. Are they better than before? Why?

4. Compare the results reported in Example 13.10 and in particular Figure 13.7 to those obtained in Question 3. Explain similarities and differences.