CPSC 303, Fall Term, 2010
Assignment 2, due Wednesday October 13th

Please show all your work: e-mail your Matlab programs to `cs303@ugrad.cs.ubc.ca`, but provide a hardcopy of the rest of the assignment. When e-mailing your programs, include your name and student ID in the message's title.

Do not e-mail a complete assignment.

1. Verify that the Hermite cubic interpolating $f(x)$ and its derivative at the points t_i and t_{i+1} can be written explicitly as

$$\begin{aligned} s_i(x) &= f_i + (h_i f_i')\,\tau + \Big(3(f_{i+1} - f_i) - h_i(f_{i+1}' + 2f_i')\Big)\tau^2 \\ &+ \Big(h_i(f_{i+1}' + f_i') - 2(f_{i+1} - f_i)\Big)\tau^3, \end{aligned}$$

where $h_i = t_{i+1} - t_i$, $f_i = f(t_i)$, $f_i' = f'(t_i)$, $f_{i+1} = f(t_{i+1})$, $f_{i+1}' = f'(t_{i+1})$, and $\tau = \frac{x - t_i}{h_i}$.

2. The *gamma function* is defined by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt, \quad x > 0.$$

It is known that for integer numbers the function has the value

$$\Gamma(n) = (n-1)! = 1 \cdot 2 \cdot 3 \cdots (n-1).$$

(We define $0! = 1$.) Thus, for example, $(1,1), (2,1), (3,2), (4,6), (5,24)$ can be used as data points for an interpolating polynomial.

   (a) Write a Matlab script that computes the polynomial interpolant of degree four that passes through the above five data points.

   (b) Write a program that computes a cubic spline to interpolate the same data. (You may use Matlab's `spline`.)

(c) Plot the two interpolants you found on the same graph, along with a plot of the gamma function itself, which can be produced using the MATLAB command `gamma`.

(d) Plot the errors in the two interpolants on the same graph. What are your observations?

3. Consider using cubic splines to interpolate the function

$$f(x) = e^{3x} \sin(200x^2)/(1 + 20x^2), \quad 0 \le x \le 1,$$

featured in Figure 10.8 of the notes.

Write a short MATLAB script using `spline`, interpolating this function at equidistant points $x_i = i/n,\ i = 0, 1, \ldots, n$. Repeat this for $n = 2^j,\ j = 4, 5, \ldots, 14$. For each such calculation record the maximum error at the points `x = 0:.001:1`. Plot these errors against n using `loglog`.

Make observations in comparison to Figure 10.8.

4. Using parametric cubic Hermite polynomials, draw a car shape design in a MATLAB graph. Your "car" view may be of the front or profile and must include some details such as wheels, headlamps, windows, etc. You may use Bezier polynomials to construct the necessary parametric curves. The graph must clearly mark the interpolation and guidepoints used for each curve. Include also a list of the interpolation and guidepoints used for each feature of the car. Finally, include a second plot which shows the curves of your car plot without the guidepoints and interpolation points.

Hints:

- You should not have to use a lot of curves to draw a crude representation of a car. At the same time, you are hereby given artistic license to do better than the bare minimum.
- Experiment with the parametric curves before you begin to construct your picture.
- You may want to write two generic MATLAB functions that can be used by `plot` to construct all of the parametric curves.

- Use the `help` command in MATLAB to check the features of `plot`, `ginput` and whatever else you find worthwhile.

A modest prize will be awarded to the student who constructs the "most beautiful" car.