CPSC 303, Fall Term, 2010
Assignment 1, due Wednesday September 29th

Please show all your work: e-mail your MATLAB programs to `cs303@ugrad.cs.ubc.ca`, but provide a hardcopy of the rest of the assignment. When e-mailing your programs, include your name and student ID in the message's title.

Do not e-mail a complete assignment.

1. The function $f_1(x_0, h) = \sin(x_0 + h) - \sin(x_0)$ can be transformed into another form, $f_2(x_0, h)$, using the trigonometric formula
$$\sin(\phi) - \sin(\psi) = 2\cos\left(\frac{\phi + \psi}{2}\right)\sin\left(\frac{\phi - \psi}{2}\right).$$
Thus, f_1 and f_2 have the same values, in exact arithmetic, for any given argument values x_0 and h.
   (a) Derive $f_2(x_0, h)$.
   (b) Suggest a formula that avoids cancellation errors for computing the approximation $(f(x_0+h)-f(x_0))/h$ to the derivative of $f(x) = \sin(x)$ at $x = x_0$. Write a MATLAB program that implements your formula and computes an approximation of $f'(1.2)$, for h =1e-20,1e-19,...,1.
   (c) Explain the difference in accuracy between your results and the results reported in Example 1.3.

2. Some unknown (but crucial) modeling considerations have mandated a search for a function
$$u(x) = \gamma_0 e^{\gamma_1 x + \gamma_2 x^2},$$
where the unknown coefficients γ_1 and γ_2 are expected to be nonpositive. Given are data pairs to be interpolated, (x_0, z_0), (x_1, z_1) and (x_2, z_2), where $z_i > 0$, $i = 0, 1, 2$. Thus, we require $u(x_i) = z_i$.
The function $u(x)$ is not linear in its coefficients, but $v(x) = \ln(u(x))$ is linear in its.

(a) Find a quadratic polynomial $v(x)$ that interpolates appropriately defined three data pairs, and then give a formula for $u(x)$ in terms of the original data.

[This is a pen-and-paper item; the following one should consume much less of your time.]

(b) Write a script to find u for the data $(0, 1)$, $(1, .9)$, $(3, .5)$. Give the coefficients γ_i and plot the resulting interpolant over $[0, 6]$. In what way does the curve behave qualitatively differently from a quadratic?

3. Joe had decided to buy stocks of a particularly promising Internet company. The price per share was \$100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired:
$(0, 100), (7, 98), (14, 101), (21, 50), (28, 51), (35, 50)$.

   In attempting to analyze what happened, it was desired to approximately evaluate the stock price a few days before the crash.

   (a) Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at $x = 12$. Which one do you think is the most accurate? Explain.

   (b) Plot the two quadratic interpolants above, together with the data (without a broken line passing through the data) over the interval $[0, 21]$. What are your observations?

4. Suppose we want to approximate the function e^x on the interval $[0, 1]$ by using polynomial interpolation with $x_0 = 0, x_1 = 1/2$ and $x_2 = 1$. Let $p_2(x)$ denote the interpolating polynomial.

   (a) Find an upper bound for the error magnitude

$$\max_{0 \le x \le 1} |e^x - p_2(x)|.$$

   (b) Find the interpolating polynomial using your favorite technique.

   (c) Plot the function e^x and the interpolant you found, both on the same figure, using the commands `plot`.

(d) Plot the error magnitude $|e^x - p_2(x)|$ on the interval using logarithmic scale (the command `semilogy`) and verify by inspection that it is below the bound you found in Part (a).

5. Interpolate the Runge function

$$f(x) = \frac{1}{1 + 25x^2}, \quad -1 \leq x \leq 1,$$

at Chebyshev points for n from 10 to 170 in increments of 10. Calculate the maximum interpolation error on the uniform evaluation mesh `x = -1:.001:1` and plot the error vs. polynomial degree using `semilogy`. Observe spectral accuracy.