Stat 302 Midterm (202)

Q1. Define:

A = Linda gets an A;

B = John gets an A.

We know that $P(A) = P(B)$, $P(A \cup B) = 0.7$, and $P(A \cap B) = 0.3$.

(a)

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 2P(A) - P(A \cap B) \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad P(A) &= \left(P(A \cup B) + P(A \cap B)\right)/2 \\ &= (0.7 + 0.3)/2 = 0.5. \end{aligned}$$

(b)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.5} = 0.6.$$

(c)

$$P(A \cap B|A \cup B) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.3}{0.7} = 0.43.$$

Q2. According the graph, we have

$$\begin{aligned} P(\text{I works}) &= P(a_1 \text{ works} \cup a_2 \text{ works}) \\ &= 1 - P(a_1 \text{ works} \cap a_2 \text{ works}) \\ &= 1 - P(a_1 \text{ works}) \times P(a_2 \text{ works}) \quad \text{(independence)} \\ &= 1 - 0.10^2. \end{aligned}$$

Similarly,

$$\begin{aligned} P(\text{II works}) &= 1 - 0.20^2, \\ P(\text{III works}) &= 1 - 0.15^2. \end{aligned}$$

$$\begin{aligned} P(\text{system works}) &= P(\text{I works} \cap \text{II works} \cap \text{III works}) \\ &= P(\text{I works}) \times P(\text{II works}) \times P(\text{III works}) \\ &= (1 - 0.10^2) \times (1 - 0.20^2) \times (1 - 0.15^2) \\ &= 0.95998. \end{aligned}$$

Q3. Define: F = there is a flaw. We have

$$P(F) = 0.005, \quad P(F^c) = 0.995.$$

$$P(+|F) = 0.999, \quad P(+|F^c) = 0.020.$$

(a)

$$\begin{aligned} P(+) &= P(+|F)P(F) + P(+|F^c)P(F^c) \\ &= 0.999 \times 0.005 + 0.020 \times 0.995 \\ &= 0.024895. \end{aligned}$$

(b)

$$\begin{aligned} P(F|-) &= \frac{P(-|F)P(F)}{P(-)} \\ &= \frac{0.001 \times 0.005}{1 - 0.024895} \\ &= 0.0000051277. \end{aligned}$$

Q4. (a)Since

$$-0.10 + 0.20 + 2c_2 + 3 \times 0.2 + 4 \times 0.1 + 5 \times 0.05 = 1.55,$$

$$0.10 + c_1 + 0.20 + c_2 + 0.2 + 0.1 + 0.05 = 1,$$

by solving above equations we obtain $c_1 = 0.25, c_2 = 0.1$.

(b) Var(X)=2.9475.

(c) $E(2X^2 + 3) = 2E(X^2) + 3 = 2(\text{Var}(X) + E(X)^2) + 3 = 13.7$.