Stat 302: Midterm
Formula Sheet

- $n! = n \times (n-1) \times \cdots \times 2 \times 1$
- ${}_nP_r = {}_p\binom{n}{r} = n \times (n-1) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$
- ${}_nC_r = {}_c\binom{n}{r} = \frac{n!}{r!(n-r)!}$
- The number of ways partitioning n distinct objects into k groups containing $n_1, n_2, \ldots, n_k$ objects, respectively, is $\frac{n!}{n_1!n_2!\cdots n_k!}$, where $\sum_{i=1}^k n_i = n$.
- For events E, F and G,
  - Commutative laws: $E \cup F = F \cup E$; $E \cap F = F \cap E$
  - Associative laws: $(E \cup F) \cup G = E \cup (F \cup G)$; $(E \cap F) \cap G = E \cap (F \cap G)$
  - Distributive laws: $(E \cap F) \cup G = (E \cup G) \cap (F \cup G)$; $(E \cup F) \cap G = (E \cap G) \cup (F \cap G)$
- For events $E_1, E_2, \ldots, E_n$,
  - DeMorgan's laws: $\left(\bigcup_{i=1}^n E_i\right)^c = \bigcap_{i=1}^n E_i^c$; $\left(\bigcap_{i=1}^n E_i\right)^c = \bigcup_{i=1}^n E_i^c$
- If events E and F are mutually exclusive, then $E \cap F = \phi$, where ϕ denotes the null set
- For mutually exclusive events $E_1, E_2, \ldots$, we have $P(\bigcup_{i=1}^\infty E_i) = \sum_{i=1}^\infty P(E_i)$
- Probability rules
  1. $P(E^c) = 1 - P(E)$
  2. If $E \subset F$, then $P(E) \leq P(F)$
  3. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
  4. $$P(\bigcup_{i=1}^n E_i) = \sum_{i=1}^n P(E_i) - \sum_{1 \leq i < j \leq n} P(E_i \cap E_j) + \sum_{1 \leq i < j < k \leq n} P(E_i \cap E_j \cap E_k) - \cdots + (-1)^{n+1} P(E_1 \cap E_2 \cap \cdots \cap E_n)$$
  5. If all outcomes in $\mathcal{S}$ are equally likely, then $P(E) = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } \mathcal{S}}$
- Conditional probability of E given F, where $P(F) > 0$,
$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$
- More about conditional probability
  1. $P(E^c|F) = 1 - P(E|F)$
  2. If $E_1, E_2, \ldots$ are mutually exclusive, then $P(\bigcup_{i=1}^\infty E_i|F) = \sum_{i=1}^\infty P(E_i|F)$
- Bayes' formula: If $F_1, F_2, \ldots, F_n$ are mutually exclusive, then
$$P(F_i|E) = \frac{P(F_i \cap E)}{P(E)} = \frac{P(E|F_i)P(F_i)}{\sum_{j=1}^n P(E|F_j)P(F_j)}$$
- Odds of event E: $\frac{P(E)}{P(E^c)}$ or $\frac{P(E)}{1-P(E)}$

- Independence of events E and F:
  1. $P(E|F) = P(E)$
  2. $P(F|E) = P(F)$
  3. $P(E \cap F) = P(E) \times P(F)$
- Conditional independence of events E and F given G:
  1. $P(E|F \cap G) = P(E|G)$
  2. $P(F|E \cap G) = P(F|G)$
  3. $P(E \cap F|G) = P(E|G) \times P(F|G)$
- Probability mass function: $p(a) = P(X = a)$
- Cumulative distribution function: $F(b) = P(X \leq b)$
- Expected value, expectation, mean: $E(X) = \sum_{\text{all x}} x\, p(x)$
- Expectation of $g(x)$: $E(g(X)) = \sum_{\text{all x}} g(x)\, p(x)$
- Variance $V(X)$ and standard deviation $SD(X) = \sqrt{V(X)}$:

$$V(X) = \sum_{\text{all x}} (x - \mu)^2 p(x) = E(X^2) - \mu^2, \text{ where } \mu = E(X)$$

- Rules about $E(X)$ and $V(X)$:

  For any real numbers a and b, real functions g_1 and g_2, and random variables X and Y,

  1. $E(aX + b) = aE(X) + b$
  2. $E(ag_1(X) + bg_2(Y)) = aE(g_1(X)) + bE(g_2(Y))$
  3. $V(aX + b) = a^2 V(X)$

- Common discrete random variables:
  1. Bernoulli: $X \sim Bernoulli(p)$
     (a) $p(x) = p^x (1-p)^{1-x}$, where $x \in \{0, 1\}$
     (b) $E(X) = p$, $V(X) = p(1-p)$
  2. Binomial: $X \sim Bin(n, p)$
     (a) $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$, where $x \in \{0, 1, 2, \ldots, n\}$
     (b) $E(X) = np$, $V(X) = np(1-p)$
  3. Geometric: $X \sim Geom(p)$
     (a) $p(x) = (1-p)^{x-1} p$, where $x \in \{1, 2, 3, \ldots\}$
     (b) $E(X) = \frac{1}{p}$, $V(X) = \frac{1-p}{p^2}$
     (c) $F(x) = 1 - (1-p)^x$
  4. Negative Binomial: $X \sim Neg.Bin(r, p)$
     (a) $p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$, where $x \in \{r, r+1, \ldots\}$
     (b) $E(X) = \frac{r}{p}$, $V(X) = \frac{r(1-p)}{p^2}$

5. Poisson: $X \sim Poisson(\lambda t)$,
   where λ is the expected number of "events" that occur **per unit time** and t is the number of time units
   (a) $p(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$, where $x \in \{0, 1, 2, \ldots, \}$
   (b) $E(X) = \lambda t$, $V(X) = \lambda t$
   (c) Poisson Approximation to the Binomial:
   If $X \sim Bin(n, p)$, n is sufficiently large, and p or $(1-p)$ is small enough such that $min(np, n(1-p)) < 5$, then
   $$X \overset{approx.}{\sim} Poisson(\lambda t = np)$$
6. Hypergeometric: $X \sim Hypergeom(N, n, m)$
   (a) $p(x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$, where $x \in \{0, 1, \ldots, n\}$
   (b) $E(X) = \frac{nm}{N}$, $V(X) = \frac{nm}{N}(1 - \frac{m}{N})(\frac{N-n}{N-1})$