STAT 302: Chapter 5 Solutions to Suggested Exercises

2. $\int xe^{-x/2}dx = -2xe^{-x/2} - 4e^{-x/2}$. Hence,

$$c\int_0^\infty xe^{-x/2}dx = 1 \Rightarrow c = 1/4$$

$$P\{X > 5\} = \frac{1}{4}\int_5^\infty xe^{-x/2}dx = \frac{1}{4}[10e^{-5/2} + 4e^{-5/2}]$$
$$= \frac{14}{4}e^{-5/2}$$

4. (a) $\int_{20}^\infty \frac{10}{x^2}dx = \frac{-10}{x}\Big|_{20}^\infty = 1/2$.

(b) $F(y) = \int_{10}^y \frac{10}{x^2}dx = 1 - \frac{10}{y}$, $y > 10$. $F(y) = 0$ for $y < 10$.

(c) $\sum_{i=3}^{6}\binom{6}{i}\left(\frac{2}{3}\right)^i\left(\frac{1}{3}\right)^{6-i}$ since $\overline{F}(15) = \frac{10}{15}$. Assuming independence of the events that the devices exceed 15 hours.

5. Must choose c so that

$$.01 = \int_c^1 5(1-x)^4 dx = (1-c)^5$$

so $c = 1 - (.01)^{1/5}$.

8. $E[X] = \int_0^\infty x^2 e^{-x}dx = \Gamma(3) = 2$

10. (a) $P\{\text{goes to } A\} = P\{5 < X < 15 \text{ or } 20 < X < 30 \text{ or } 35 < X < 45 \text{ or } 50 < X < 60\}$.
$= 2/3$ since X is uniform (0, 60).

(b) same answer as in (a).

15. (a) $\Phi(.8333) = .7977$
(b) $2\Phi(1) - 1 = .6827$
(c) $1 - \Phi(.3333) = .3695$
(d) $\Phi(1.6667) = .9522$
(e) $1 - \Phi(1) = .1587$

16. $P\{X > 50\} = P\left\{\frac{X-40}{4} > \frac{10}{4}\right\} = 1 - \Phi(2.5) = 1 - .9938$

Hence, $(P\{X < 50\})^{10} = (.9938)^{10}$

17. $E[\text{Points}] = 10(1/10) + 5(2/10) + 3(2/10) = 2.6$

19. Letting $Z = (X - 12)/2$ then Z is a standard normal. Now, $.10 = P\{Z > (c - 12)/2\}$. But from Table 5.1, $P\{Z < 1.28\} = .90$ and so

$$(c - 12)/2 = 1.28 \quad \text{or} \quad c = 14.56$$

20. Let X denote the number in favor. Then X is binomial with mean 65 and standard deviation $\sqrt{65(.35)} \approx 4.77$. Also let Z be a standard normal random variable.

(a) $P\{X \geq 50\} = P\{X \geq 49.5\} = P\{X - 65\}/4.77 \geq -15.5/4.77$
$\approx P\{Z \geq -3.25\} \approx .9994$

(b) $P\{59.5 \leq X \leq 70.5\} \approx P\{-5.5/4.77 \leq Z \leq 5.5/4.77\}$
$= 2P\{Z \leq 1.15\} - 1 \approx .75$

(c) $P\{X \leq 74.5\} \approx P\{Z \leq 9.5/4.77\} \approx .977$

22. (a) $P\{.9000 - .005 < X < .9000 + .005\}$

$$= P\left\{-\frac{.005}{.003} < Z < \frac{.005}{.003}\right\}$$
$$= P\{-1.67 < Z < 1.67\}$$
$$= 2\Phi(1.67) - 1 = .9050.$$

Hence 9.5 percent will be defective (that is each will be defective with probability $1 - .9050 = .0950$).

(b) $P\left\{-\frac{.005}{\sigma} < Z < \frac{.005}{\sigma}\right\} = 2\Phi\left(\frac{.005}{\sigma}\right) - 1 = .99$ when

$$\Phi\left(\frac{.005}{\sigma}\right) = .995 \Rightarrow \frac{.005}{\sigma} = 2.575 \Rightarrow \sigma = .0019\,.$$

23. (a) $P\{149.5 < X < 200.5\} = P\left\{\frac{149.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}} < Z < \frac{200.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}}\right\}$

$$= \Phi\left(\frac{200.5 - 166.7}{\sqrt{5000/36}}\right) - \Phi\left(\frac{149.5 - 166.7}{\sqrt{5000/36}}\right)$$

$$\approx \Phi(2.87) + \Phi(1.46) - 1 = .9258.$$

(b) $P\{X < 149.5\} = P\left\{Z < \frac{149.5 - 800(1/5)}{\sqrt{800\frac{1}{5}\frac{4}{5}}}\right\}$

$$= P\{Z < -.93\}$$
$$= 1 - \Phi(.93) = .1762.$$

25. Let X denote the number of unacceptable items among the next 150 produced. Since X is a binomial random variable with mean 150(.05) = 7.5 and variance 150(.05)(.95) = 7.125, we obtain that, for a standard normal random variable Z.

$$P\{X \le 10\} = P\{X \le 10.5\}$$
$$= P\left\{\frac{X - 7.5}{\sqrt{7.125}} \le \frac{10.5 - 7.5}{\sqrt{7.125}}\right\}$$
$$\approx P\{Z \le 1.1239\}$$
$$= .8695$$

The exact result can be obtained by using the text diskette, and (to four decimal places) is equal to .8678.

28. Let X equal the number of lefthanders. Assuming that X is approximately distributed as a binomial random variable with parameters $n = 200$, $p = .12$, then, with Z being a standard normal random variable,

$$P\{X > 19.5\} = P\left\{\frac{X - 200(.12)}{\sqrt{200(.12)(.88)}} > \frac{19.5 - 200(.12)}{\sqrt{200(.12)(.88)}}\right\}$$
$$\approx P\{Z > -.9792\}$$
$$\approx .8363$$

32. (a) e^{-1}
(b) $e^{-1/2}$

33. e^{-1}

34. (a) $P\{X > 20\} = e^{-1}$

(b) $P\{X > 30 \mid X > 10 = \frac{P\{X > 30\}}{P\{X > 10\}} = \frac{1/4}{3/4} = 1/3$

37. (a) $P\{|X| > 1/2\} = P\{X > 1/2\} + P\{X < -1/2\} = 1/2$

(b) $P\{|X| \le a\} = P\{-a \le X \le a\} = a, 0 < a < 1$. Therefore,

$$f_{|X|}(a) = 1,\ 0 < a < 1$$

That is, $|X|$ is uniform on (0, 1).

39. $F_Y(y) = P\{\log X \le y\}$

$= P\{X \le e^y\} = F_X(e^y)$

$f_Y(y) = f_X(e^y)e^y = e^y e^{-e^y}$

40. $F_Y(y) = P\{e^X \le y\}$

$= F_X(\log y)$

$f_Y(y) = f_X(\log y)\frac{1}{y} = \frac{1}{y},\ 1 < y < e$

Theoretical Exercises:

9. The final step of parts (a) and (b) use that $-Z$ is also a standard normal random variable.

(a) $P\{Z > x\} = P\{-Z < -x\} = P\{Z < -x\}$

(b) $P\{|Z| > x\} = P\{Z > x\} + P\{Z < -x\} = P\{Z > x\} + P\{-Z > x\}$

$= 2P\{Z > x\}$

(c) $P\{|Z| < x\} = 1 - P\{|Z| > x\} = 1 - 2P\{Z > x\}$ by (b)

$= 1 - 2(1 - P\{Z < x\})$

12. (a) $\frac{b+a}{2}$

(b) μ

(c) $1 - e^{-\lambda m} = 1/2$ or $m = \frac{1}{\lambda}\log 2$

13. (a) all values in (a, b)

(b) μ

(c) 0

29. $F_Y(x) = P\{aX + b \le x\}$

$$= P\left\{X \le \frac{x-b}{a}\right\} \text{ when } a > 0$$

$$= F_X((x-b)/a) \text{ when } a > 0.$$

$$f_Y(x) = \frac{1}{a} f_X((x-b)/a) \quad \text{if } a > 0.$$

When $a < 0$, $F_Y(x) = P\left\{X \ge \frac{x-b}{a}\right\} = 1 - F_X\left(\frac{x-b}{a}\right)$ and so

$$f_Y(x) = -\frac{1}{a} f_X\left(\frac{x-b}{a}\right).$$