Stat 302 Winter 07/08 – Solutions to Suggested Problems: Chapter 3

5. $\frac{6}{15}\frac{5}{14}\frac{9}{13}\frac{8}{12}$

12. (a) $(.9)(.8)(.7) = .504$

(b) Let F_i denote the event that she failed the ith exam.

$$P(F_2 \mid (F_1^c F_2^c F_3^c)^c) = \frac{P(F_1^c F_2)}{1-.504} = \frac{(.9)(.2)}{.496} = .3629$$

16. With S being survival and C being C section of a randomly chosen delivery, we have that

$$.98 = P(S) = P(S \mid C).15 + P(S \mid C^2)\ .85$$
$$= .96(.15) + P(S \mid C^2)\ .85$$

Hence

$$P(S \mid C^c) \approx .9835.$$

18. (a) $P(\text{Ind} \mid \text{voted}) = \dfrac{P(\text{voted} \mid \text{Ind})P(\text{Ind})}{\sum P(\text{voted} \mid \text{type})P(\text{type})}$

$$= \frac{.35(.46)}{.35(.46) + .62(.3) + .58(.24)} \approx 331$$

(b) $P\{\text{Lib} \mid \text{voted}\} = \dfrac{.62(.30)}{.35(.46) + .62(.3) + .58(.24)} \approx .383$

(c) $P\{\text{Con} \mid \text{voted}\} = \dfrac{.58(.24)}{.35(.46) + .62(.3) + .58(.24)} \approx .286$

(d) $P\{\text{voted}\} = .35(.46) + .62(.3) + .58(.24) = .4862$
That is, 48.62 percent of the voters voted.

26. Let M be the event that the person is male, and let C be the event that he or she is color blind. Also, let p denote the proportion of the population that is male.

$$P(M \mid C) = \frac{P(C \mid M)P(M)}{P(C \mid M)P(M) + P(C \mid M^c)P(M^c)} = \frac{(.05)p}{(.05)p + (.0025)(1-p)}$$

28. Let A denote the event that the next card is the ace of spades and let B be the event that it is the two of clubs.

(a) $P\{A\} = P\{\text{next card is an ace}\}P\{A \mid \text{next card is an ace}\}$

$$= \frac{3}{32}\frac{1}{4} = \frac{3}{128}$$

(b) Let C be the event that the two of clubs appeared among the first 20 cards.

$$P(B) = P(B \mid C)P(C) + P(B \mid C^c)P(C^c)$$
$$= 0\frac{19}{48} + \frac{1}{32}\frac{29}{48} = \frac{29}{1536}$$

33. Let V be the event that the letter is a vowel. Then

$$P(E \mid V) = \frac{P(V|E)P(E)}{P(V|E)P(E) + P(V|A)P(A)} = \frac{(1/2)(2/5)}{(1/2)(2/5) + (2/5)(3/5)} = 5/11$$

37. (a) $P\{\text{fair} \mid h\} = \dfrac{\frac{1}{2}\frac{1}{2}}{\frac{1}{2}\frac{1}{2} + \frac{1}{2}} = \dfrac{1}{3}$.

(b) $P\{\text{fair} \mid hh\} = \dfrac{\frac{1}{4}\frac{1}{2}}{\frac{1}{4}\frac{1}{2} + \frac{1}{2}} = \dfrac{1}{5}$.

(c) 1

48. (a) $P\{\text{silver in other} \mid \text{silver found}\}$

$$= \frac{P\{S \text{ in other, } S \text{ found}\}}{P\{S \text{ found}\}}.$$

To compute these probabilities, condition on the cabinet selected.

$$= \frac{1/2}{P\{S \text{ found}|A\}1/2 + P\{S \text{ found}|B\}1/2}$$
$$= \frac{1}{1 + 1/2} = \frac{2}{3}.$$

57. (a) $2p(1-p)$

(b) $\binom{3}{2}p^2(1-p)$

(c) $P\{\text{up on first} \mid \text{up 1 after 3}\}$
$= P\{\text{up first, up 1 after 3}\}/[3p^2(1-p)]$
$= p2p(1-p)/[3p^2(1-p)] = 2/3.$

61. Because the non-albino child has an albino sibling we know that both its parents are carriers. Hence, the probability that the non-albino child is not a carrier is

$$P(A, A \mid A, a \text{ or } a, A \text{ or } A, A) = \frac{1}{3}$$

Where the first gene member in each gene pair is from the mother and the second from the father. Hence, with probability 2/3 the non-albino child is a carrier.

(a) Condition on whether the non-albino child is a carrier. With C denoting this event, and O_i the event that the i^{th} offspring is albino, we have:

$$P(O_1) = P(O_1 \mid C)P(C) + P(O_1 \mid C^c)P(C^c)$$
$$= (1/4)(2/3) + 0(1/3) = 1/6$$

(b) $$P(O_2 \mid O_1^c) = \frac{P(O_1^c O_2)}{P(O_1^c)}$$
$$= \frac{P(O_1^c O_2 \mid C)P(C) + P(O_1^c O_2 \mid C^c)P(C^c)}{5/6}$$
$$= \frac{(3/4)(1/4)(2/3) + 0(1/3)}{5/6} = \frac{3}{20}$$

73. (a) 1/16, (b) 1/32, (c) 10/32, (d) 1/4, (e) 31/32.

Theoretical Ex.

2. If $A \subset B$

$$P(A \mid B) = \frac{P(A)}{P(B)},\ P(A \mid B^c) = 0, \qquad P(B \mid A) = 1, \qquad P(B \mid A^c) = \frac{P(BA^c)}{P(A^c)}$$

4. Let N_i denote the event that the ball is not found in a search of box i, and let B_j denote the event that it is in box j.

$$P(B_j \mid N_i) = \frac{P(N_i \mid B_j)P(B_j)}{P(N_i \mid B_i)P(B_i) + P(N_i \mid B_i^c)P(B_i^c)}$$

$$= \frac{P_j}{(1-\alpha_i)P_i + 1 - P_i} \text{ if } j \neq i$$

$$= \frac{(1-\alpha_i)P_i}{(1-\alpha_i)P_i + 1 - P_i} \text{ if } j = i$$

25. $P(E \mid F) = \text{P}(EF)/P(F)$

$$P(E \mid FG)P(G \mid F) = \frac{P(EFG)}{P(FG)}\frac{P(FG)}{P(F)} = \frac{P(EFG)}{P(F)}$$

$$P(E \mid FG^c)P(G^c \mid F) = \frac{P(EFG^c)}{P(F)}\,.$$

The result now follows since

$$P(EF) = P(EFG) + P(EFG^c)$$