Stat 302 Winter 07/08 – Solutions to Suggested Problems: Chapter 1

4. There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are $2 \cdot 1 \cdot 2 \cdot 1 = 4$ possibilities.

6. Each kitten can be identified by a code number i, j, k, l where each of i, j, k, l is any of the numbers from 1 to 7. The number i represents which wife is carrying the kitten, j then represents which of that wife's 7 sacks contain the kitten; k represents which of the 7 cats in sack j of wife i is the mother of the kitten; and l represents the number of the kitten of cat k in sack j of wife i. By the generalized principle there are thus $7 \cdot 7 \cdot 7 \cdot 7 = 2401$ kittens

8. (a) $5! = 120$

(b) $\frac{7!}{2!2!} = 1260$

(c) $\frac{11!}{4!4!2!} = 34{,}650$

(d) $\frac{7!}{2!2!} = 1260$

11. (a) 6!

(b) 3!2!3!

(c) 3!4!

13. $\binom{20}{2}$

18. $\binom{5}{2}\binom{6}{2}\binom{4}{3} = 600$

19. (a) There are $\binom{8}{3}\binom{4}{3} + \binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees.

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

(b) There are $\binom{6}{3}\binom{6}{3} + \binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees.

(c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ possible committees. There are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves; $\binom{7}{2}\binom{5}{3}$ in which the feuding women serves; and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves.

21. $\frac{7!}{3!4!} = 35$. Each path is a linear arrangement of 4 *r*'s and 3 *u*'s (*r* for right and *u* for up). For instance the arrangement *r*, *r*, *u*, *u*, *r*, *r*, *u* specifies the path whose first 2 steps are to the right, next 2 steps are up, next 2 are to the right, and final step is up.

22. There are $\frac{4!}{2!2!}$ paths from A to the circled point; and $\frac{3!}{2!1!}$ paths from the circled point to B. Thus, by the basic principle, there are 18 different paths from A to B that go through the circled piont.

23. $3!2^3$

Theoretical ex.

10. Parts (a), (b), (c), and (d) are immediate. For part (e), we have the following:

$$k\binom{n}{k} = \frac{k!n!}{(n-k)!k!} = \frac{n!}{(n-k)!(k-1)!}$$

$$(n-k+1)\binom{n}{k-1} = \frac{(n-k+1)n!}{(n-k+1)!(k-1)!} = \frac{n!}{(n-k)!(k-1)!}$$

$$n\binom{n-1}{k-1} = \frac{n(n-1)!}{(n-k)!(k-1)!} = \frac{n!}{(n-k)!(k-1)!}$$