

## Stat 302 Assignment 3

Please remember to **INCLUDE A COVER SHEET** when you submit your assignment. It is due on Thursday, 15th April at 5pm in the designated drop off boxes next to the Statistics general office in the LSK building. When answering the questions, writing down the final answer will not be sufficient to receive full marks. Please show all calculations unless otherwise specified.

Q1. (10) Suppose that 3 balls are randomly selected from an urn containing 3 red, 4 white, and 5 blue balls. Let  $X$  and  $Y$  denote, respectively, the number of red and white balls chosen.

- Find the joint probability mass function of  $X$  and  $Y$ .
- Determine  $P(X = Y)$ ,  $P(X > Y)$ ,  $P(X + Y > 2)$ ,  $P(XY \leq 3)$ .
- Find the marginal probability mass functions of  $X$  and  $Y$ .
- Find the conditional probability mass functions of  $X$  when (i)  $Y = 1$ ; (ii)  $Y = 2$ .

Q2. (10) The joint probability density function of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{1}{8}(y^2 - x^2)e^{-y}, \quad -y \leq x \leq y, 0 < y < \infty.$$

- Are  $X$  and  $Y$  independent?
- Find the marginal pdfs of  $X$  and  $Y$ .
- Find the conditional pdf of  $X$ , given  $Y = y$ .
- Find the conditional distribution function of  $X$ , given  $Y = y$ . Evaluate the probability of  $X > 1$  under the conditions (i)  $Y = 2$ ; (ii)  $Y = 3$ .

Q3. (10) Let  $X$  and  $Y$  be random variables with joint pdf

$$f(x, y) = \begin{cases} 1/2 & \text{for } x > 0, y > 0, x + y < 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal pdfs of  $X$  and  $Y$ .
- Determine whether the two random variables are independent.

(c) Calculate  $P(X > 2Y)$ .

Q4. Let  $X$  and  $Y$  be independent random variables with marginal pdfs

$$f_X(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}, \quad f_Y(y) = \begin{cases} e^{-y} & \text{for } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine  $P(X < 3Y)$ .

(b) Find the pdf of  $Z = X + Y$ .

Q5. Suppose that  $X$  and  $Y$  have a continuous joint distribution for which the joint pdf is defined as follows:

$$f(x, y) = \begin{cases} cy^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

Determine

(a) the value of  $c$ ,

(b)  $P(X + Y > 2)$ ,

(c)  $P(Y < 1/2)$ ,

(d)  $P(X \leq 1)$ ,

(e)  $P(X = 3Y)$ .

Q6. Let  $X$  and  $Y$  be two random variables whose joint pdf is given by

$$f(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)}\right\}$$

(a) Show that  $X$  and  $Y$  are both standard normal random variables.

(b) Show that the mean of the conditional pdf  $f_{X|Y}(x|y)$  is  $E(X|Y = y) = \rho y$ .

(c) Show that the variance of the conditional pdf  $f_{X|Y}(x|y)$  is  $Var(X|Y = y) = (1 - \rho^2)$ .