Stat 302 Assignment 1

Please remember to **INCLUDE A COVER SHEET** when you submit your assignment. It is due on Thursday, 11th February at 5pm in the designated drop off boxes next to the Statistics general office in the LSK building. When answering the questions, writing down the final answer will not be sufficient to receive full marks. Please show all calculations unless otherwise specified.

Q1 (15 marks). There are two red flags ($R1, R2$) two blue flags ($B1, B2$) and three green flags ($G1, G2, G3$). Assume that flags of the same color are distinguishable. How many different signals (sequences of these 7 flags ($R1, R2, B1, B2, G1, G2, G3$)) can be made if

(a) there are no restrictions?
(b) the flags with the same color must be put together?
(c) flags $R1, B1$ and $G1$ must be put together?
(d) $R2$ and $B2$ cannot be put together?

Q2 (9 marks). Consider the same scenario as in Question 1. However we now assume that flags of the same color are indistinguishable. How many different signals can be made if

(a) there are no restrictions?
(b) the flags with the same color must be put together?
(c) a red, blue and green flags must be put together?

Q3 (12 marks). Jack bought 12 bottles of beer of 300ml each, 4 out of which are unqualified (contain less volume than 300ml). Jack randomly selects 6 bottles, what is the probability that

(a) all 6 bottles are qualified?
(b) at least 5 bottles are qualified?
(c) at most 3 bottles are qualified?
(d) at least 1 bottle is unqualified?

Q4 (9 marks). Given 20 people, what is the probability that among the 12 months in the year there are 2 months containing exactly 4 birthdays each and there are 6 months containing exactly 2 birthdays each? Assume that each month is equally likely and birthdays are independent.

Q5 (15 marks). A fresh graduate is hunting for a job. The probability of getting an offer during her first interview is 20%, and this probability will increase by 10% per interview, up to a maximum of 50%. That is to say, for instance, the probability that she will have an offer is 30% at the second interview and 40% at third the interview.

(a) What is the probability that the graduate will get her first offer at the third interview?

(b) What is the probability that the graduate will have at least one offer during her three first interviews?

Q6 (12 marks). Four machines are in operation. Machine 1 produces five percent defective items, machine 2 three percent, machine 3 four percent, and machine 4 eight percent. A machine is randomly selected and an item of this machine is randomly chosen.

(a) Given that the item is defective, what is the probability that it came from machine 2?

(b) Given that the item is defective, what is the probability that it did not come from machine 2 or machine 4?

(c) Given that the item is not defective, what is the probability that it came from machine 2?

Q7 (18 marks).

(a) Prove that if $E_1, E_2, \ldots, E_n$ are independent events, then:

$$P(E_1 \cup E_2 \cup \ldots \cup E_n) = 1 - \prod_{i=1}^{n} [1 - P(E_i)].$$

(b) In a roll of a pair of dice (one red and one green), let A be the event "red die shows a 3, 4, or 5"; let B be the event "green die shows a 1 or 2"; let C be the event "dice total is 7." Show that the events A, B and C are independent.

(c) What is the probability that at least one of the events A, B, C occur.

Q8 (10 marks). The probability of UBC students infected by the H1N1 virus is 1/1000. We have a test which has 0.98 chance to detect true positive result and 0.00001 chance of yielding false positive result.

(a) What is the probability that a UBC student tests positive?

(b) What is the probability that a UBC student is infected by H1N1 given that the test result is positive?

(c) What is the probability that a UBC student is healthy given that the test result is negative?