Heuristic Search

- Idea: don't ignore the goal when selecting paths.
- Often there is extra knowledge that can be used to guide the search: heuristics.
- $h(n)$ is an estimate of the cost of the shortest path from node n to a goal node.
- $h(n)$ uses only readily obtainable information (that is easy to compute) about a node.
- h can be extended to paths: $h(\langle n_0, \ldots, n_k \rangle) = h(n_k)$.
- $h(n)$ is an underestimate if there is no path from n to a goal that has path length less than $h(n)$.

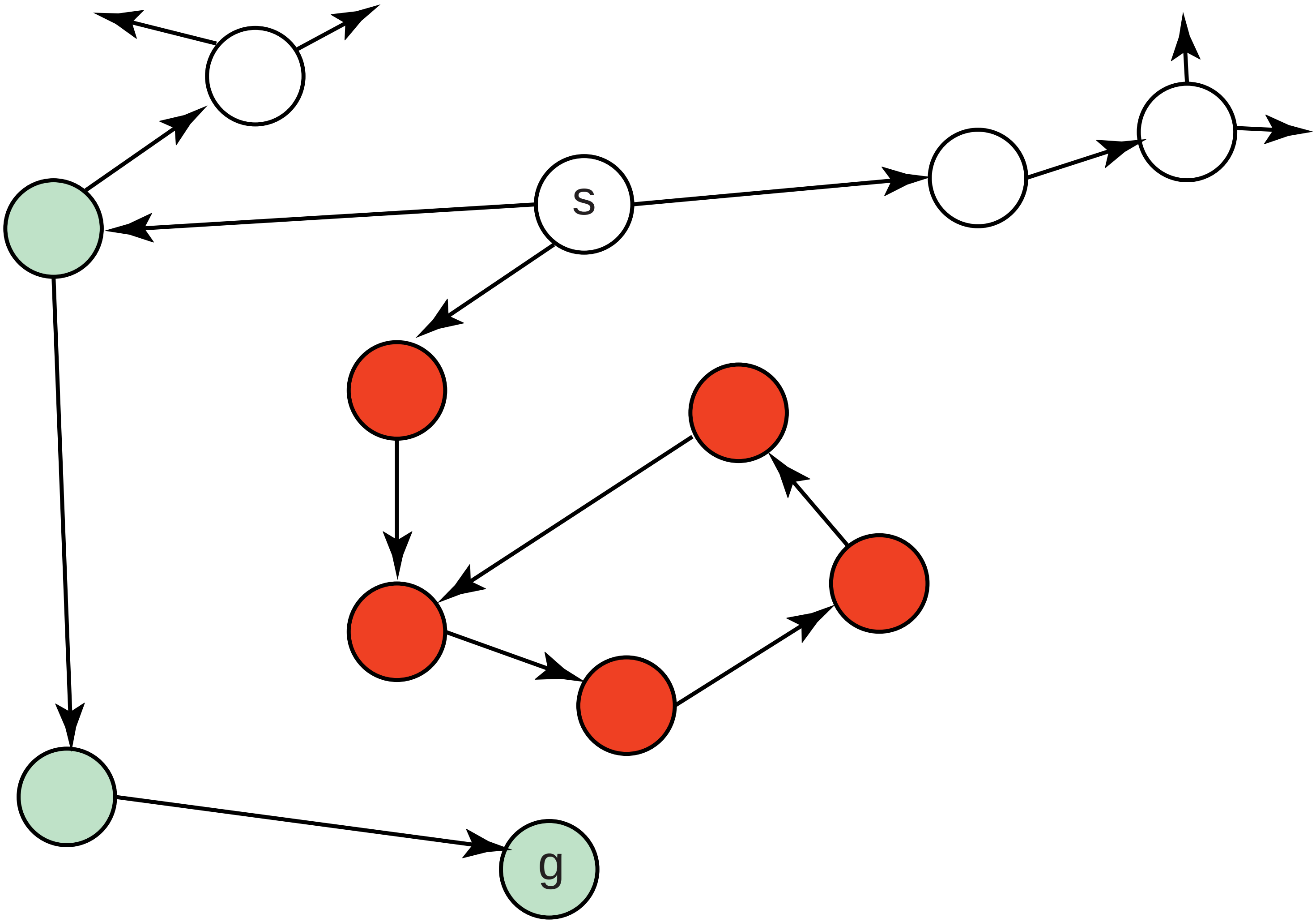
Example Heuristic Functions

- If the nodes are points on a Euclidean plane and the cost is the distance, we can use the straight-line distance from n to the closest goal as the value of $h(n)$.
- If the graph is one of queries for a derivation from a KB, one heuristic function is the number of atoms in the query.
- If the nodes are locations and cost is time, we can use the distance to a goal divided by the maximum speed.

Best-first Search

- **Idea:** select the path whose end is closest to a goal according to the heurstic function.
- Best-first search selects a path on the frontier with minimal h-value.
- It treats the frontier as a priority queue ordered by h.

Illustrative Graph — Best-first Search



Complexity of Best-first Search

- It uses space exponential in path length.
- It isn't guaranteed to find a solution, even of one exists.
- It doesn't always find the shortest path.

Heuristic Depth-first Search

- It's a way to use heuristic knowledge in depth-first search.
- Idea: order the neighbors of a node (by h) before adding them to the front of the frontier.
- It locally selects which subtree to develop, but still does depth-first search. It explores all paths from the node at the head of the frontier before exploring paths from the next node.
- Space is linear in path length. It isn't guaranteed to find a solution. It can get led up the garden path.

A^* Search

- A^* search uses both path cost and heuristic values
- $cost(p)$ is the cost of the path p.
- $h(p)$ estimates of the cost from the end of p to a goal.
- Let $f(p) = cost(p) + h(p)$. $f(p)$ estimates of the the total path cost of going from a start node to a goal via p.

$$\underbrace{\underbrace{start \xrightarrow{\text{path } p} n}_{cost(p)} \underbrace{\xrightarrow{\text{estimate}} goal}_{h(n)}}_{f(p)}$$

A^* Search Algorithm

- A^* is a mix of lowest-cost-first and best-first search.
- It treats the frontier as a priority queue ordered by $f(n)$.
- It always selects the node on the frontier with the lowest estimated distance from the start to a goal node constrained to go via that node.

Admissibility of A^*

If there is a solution, A^* always finds an optimal solution —the first path to a goal selected— if

- the branching factor is finite
- arc costs are bounded above zero (there is some $\epsilon > 0$ such that all of the arc costs are greater than ϵ), and
- $h(n)$ is an underestimate of the length of the shortest path from n to a goal node.

Why is A^* admissible?

- If a path p to a goal is selected from a frontier, can there be a shorter path to a goal?
- Suppose path p' is on the frontier. Because p was chosen before p', and $h(p) = 0$:

$$cost(p) \leq cost(p') + h(p').$$

- Because h is an underestimate

$$cost(p') + h(p') \leq cost(p'')$$

  for any path p'' to a goal that extends p'
- So $cost(p) \leq cost(p'')$ for any other path p'' to a goal.

Why is A^* admissible?

- There is always an element of an optimal solution path on the frontier before a goal has been selected. This is because, in the abstract search algorithm, there is the initial part of every path to a goal.

- A^* halts, as the minimum g-value on the frontier keeps increasing, and will eventually exceed any finite number.