Proofs

- A **proof** is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB.
- Recall $KB \models g$ means g is true in all models of KB.
- A proof procedure is **sound** if $KB \vdash g$ implies $KB \models g$.
- A proof procedure is **complete** if $KB \models g$ implies $KB \vdash g$.

Bottom-up Ground Proof Procedure

One rule of derivation, a generalized form of *modus ponens*:

> If "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" is a clause in the knowledge base, and each b_i has been derived, then h can be derived.

You are forward chaining on this clause.

(This rule also covers the case when $m = 0$.)

Bottom-up proof procedure

$KB \vdash g$ if $g \in C$ at the end of this procedure:

$C := \{\}$;

repeat

select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in KB such that

$b_i \in C$ for all i, and

$h \notin C$;

$C := C \cup \{h\}$

until no more clauses can be selected.

Example

$$a \leftarrow b \wedge c.$$

$$a \leftarrow e \wedge f.$$

$$b \leftarrow f \wedge k.$$

$$c \leftarrow e.$$

$$d \leftarrow k.$$

$$e.$$

$$f \leftarrow j \wedge e.$$

$$f \leftarrow c.$$

$$j \leftarrow c.$$

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.

Let h be the first atom added to C that's not true in every model of KB. Suppose h isn't true in model I of KB. There must be a clause in KB of form

$$h \leftarrow b_1 \wedge \ldots \wedge b_m$$

Each b_i is true in I. h is false in I. So this clause is false in I. Therefore I isn't a model of KB.

Contradiction: thus no such g exists.

Fixed Point

The C generated at the end of the bottom-up algorithm is called a fixed point.

Let I be the interpretation in which every element of the fixed point is true and every other atom is false.

I is a model of KB.

Proof: suppose $h \leftarrow b_1 \wedge \ldots \wedge b_m$ in KB is false in I. Then h is false and each b_i is true in I. Thus h can be added to C. Contradiction to C being the fixed point.

I is called a Minimal Model.

Completeness

If $KB \models g$ then $KB \vdash g$.

Suppose $KB \models g$. Then g is true in all models of KB.

Thus g is true in the minimal model.

Thus g is generated by the bottom up algorithm.

Thus $KB \vdash g$.