Semantics: General Idea

A **semantics** specifies the meaning of sentences in the language.

An **interpretation** specifies:

- what objects (individuals) are in the world
- the correspondence between symbols in the computer and objects & relations in world
  - constants denote individuals
  - predicate symbols denote relations

Formal Semantics

An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- D, the domain, is a nonempty set. Elements of D are individuals.
- ϕ is a mapping that assigns to each constant an element of D. Constant c denotes individual $\phi(c)$.
- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{\textit{TRUE}, \textit{FALSE}\}$.

Example Interpretation

Constants: *phone*, *pencil*, *telephone*.

Predicate Symbol: *noisy* (unary), *left_of* (binary).

- $D = \{$✂, ☎, ✎$\}$.
- $\phi(phone) =$ ☎, $\phi(pencil) =$ ✎, $\phi(telephone) =$ ☎.
- $\pi(noisy)$:

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

$\pi(left\_of)$:

⟨✂, ✂⟩	*FALSE*	⟨✂, ☎⟩	*TRUE*	⟨✂, ✎⟩	*TRUE*
⟨☎, ✂⟩	*FALSE*	⟨☎, ☎⟩	*FALSE*	⟨☎, ✎⟩	*TRUE*
⟨✎, ✂⟩	*FALSE*	⟨✎, ☎⟩	*FALSE*	⟨✎, ✎⟩	*FALSE*

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.
- $\pi(p)$ specifies whether the relation denoted by the n-ary predicate symbol p is true or false for each n-tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.

Truth in an interpretation

A constant c denotes in I the individual $\phi(c)$.

Ground (variable-free) atom $p(t_1, \ldots, t_n)$ is

- true in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) = \mathit{TRUE}$, where t_i denotes t'_i in interpretation I and
- false in interpretation I if $\pi(p)(t'_1, \ldots, t'_n) = \mathit{FALSE}$.

Ground clause $h \leftarrow b_1 \wedge \ldots \wedge b_m$ is false in interpretation I if h is false in I and each b_i is true in I, and is true in interpretation I otherwise.

Example Truths

In the interpretation given before:

$noisy(phone)$	true
$noisy(telephone)$	true
$noisy(pencil)$	false
$left\_of(phone, pencil)$	true
$left\_of(phone, telephone)$	false
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	true
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	false
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	true

Models and logical consequences

- A knowledge base, KB, is true in interpretation I if and only if every clause in KB is true in I.
- A **model** of a set of clauses is an interpretation in which all the clauses are true.
- If KB is a set of clauses and g is a conjunction of atoms, g is a **logical consequence** of KB, written $KB \models g$, if g is true in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is true and g is false.

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	$\pi(p)$	$\pi(q)$	$\pi(r)$	$\pi(s)$	
I_1	*TRUE*	*TRUE*	*TRUE*	*TRUE*	is a model of *KB*
I_2	*FALSE*	*FALSE*	*FALSE*	*FALSE*	not a model of *KB*
I_3	*TRUE*	*TRUE*	*FALSE*	*FALSE*	is a model of *KB*
I_4	*TRUE*	*TRUE*	*TRUE*	*FALSE*	is a model of *KB*
I_5	*TRUE*	*TRUE*	*FALSE*	*TRUE*	not a model of *KB*

$KB \models p$, $KB \models q$, $KB \not\models r$, $KB \not\models s$

User's view of Semantics

1. Choose a task domain: intended interpretation.

2. Associate constants with individuals you want to name.

3. For each relation you want to represent, associate a predicate symbol in the language.

4. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.

5. Ask questions about the intended interpretation.

6. If $KB \models g$, then g must be true in the intended interpretation.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.